

A methodology for correcting refraction in vertical angles for precise monitoring in tunnels

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ABSTRACT

During the last three decades, the possibility of the high-precision instruments has been offered to the geodetic community by the technological development. Despite of this advantage some crucial parameters affect their performance. One of these parameters is the refraction.

The geodetic refraction mainly affects the zenith (vertical) angle measurements, especially in tunnels and underground facilities, where high accuracy is needed for the monitoring of the deformations. The term atmospheric or geodetic refraction, is found in the relevant surveying literature as a mean to describe the alteration in the direction of the light curve as it propagates through the different layers of the lower part of the Earth's atmosphere.

In this present work, a new methodology is analyzed in order to eliminate the influence of the geodetic refraction in the zenith angle measurements. The main idea is the accurate measurement of the air temperature in different heights in order to calculate the temperature gradient at each position, where measurements of angles took place.

The application of the method took place through the analysis of the adjustment results of 3D geodetic network, which has been implemented in the TT1 tunnel at CERN. The choice of this test field is connected with the existence of seven Hydrostatic Levelling Systems (HLS) in the TT1 tunnel. These systems can provide height differences which are unaffected by the refraction with accuracy of $\pm 10 \mu\text{m}$. This choice permits the check of the results.

Finally, after the analysis of the results the new methodology is proved to be adequate for such accurate measurements since the standard deviation of the zenith angles residuals in 3D network adjustment is reduced approximately 70% after the refraction corrections and approaches the specifications of laser tracker ($\pm 1.5 \text{ cc}$). Additionally, the maximum difference between the nominal height differences of HLS systems and the calculated height differences after the 3D network adjustment with the corrected zenith angles is very promising and approaches the value of $50 \mu\text{m}$.

I. INTRODUCTION

Refraction is called the bending of light as it passed from one medium into another. The explanation of the phenomenon is counting on the Fermat's principle and Snell's law (Arabatzi 2007; Jenkins et al., 2001; Lambrou et al., 2010; Law et al., 2015). According to these, a ray changes directions as it travels from a medium of one refractive index to another medium that has a different refractive index in order to follow the quickest path. The same effect happens when a ray travels into the atmosphere, because it passes from air layers with different temperatures (with different refractive indices). Therefore, the refraction has an effect in the most common surveyor's measurements such as in the vertical angles (trigonometric levelling, 3D networks) as well as in the geometric levelling.

In surveyor's world, the quantification of the refraction is expressed by the term of the refraction

coefficient k . The refraction coefficient is defined as the ratio of radius of earth R to the radius of the curvature of the light path ρ (Torge 2001).

Under normal atmospheric conditions and under the consideration that $k = 0.14$, the correction of geodetic refraction is calculated as one of seventh (1/7) of the correction of the curvature of earth (Shofield 1984; Tsoulis et al., 2008).

During the years, several proposals have been applied in order to determine the refraction coefficient or to eliminate its effect (Flach 2000). The most common methods are the following:

- a. The measurement of the temperature gradient which is the main influential factor for the refractive angle. (Gottwald 1985; Wilhelm 1993; Hennes et al., 1999)

- b. Incorporation of atmospheric effects into the adjustment process of geodetic networks (Elmiger 1983; Brunner 1984)
- c. Special measuring procedures such as mutual – simultaneous observations or symmetrical observation configurations (Jordan et al., 1956; Bahnert 1986)
- d. Dual – wavelength methods utilizing atmospheric dispersion to derive the refraction angle from the dispersion angle (Hertzprung 1912; Ingensand et al., 1997)
- e. Turbulent transfer model using the upward sensible heat flux for derivation of the temperature gradient (Brunner et al., 1977)

In this research work, the first method of the temperature gradient has been applied.

II. THEORETICAL ANALYSIS

A. General

The optical ray is curved as it travels into the atmosphere. The various atmospheric layers of air have different refractive indices because they have different temperature.

When an incident ray travels from a denser air layer (Figure 1), it will be refracted away from the normal $n_1 > n_2$. Whereas, if it travels from a less dense air layer to a denser one, it will be refracted towards the normal $n_1 < n_2$ (Figure 2).

Therefore, in Figure 1, the sign of the correction angle is negative (eq. 1) (negative temperature gradient) and in Figure 2, the sign of the correction angle is positive (eq. 2) (positive temperature gradient).

The second case is the same as when someone observes a fish in the water. In this case, the fish (target) is observed in a higher position than it really is.

The equations that determine the real zenith angle with respect to the measured zenith angles free of the effect of the refraction for these two cases are the following:

$$V_{Z_T} = V_{Z_m} - r \quad (1)$$

$$V_{Z_T} = V_{Z_m} + r \quad (2)$$

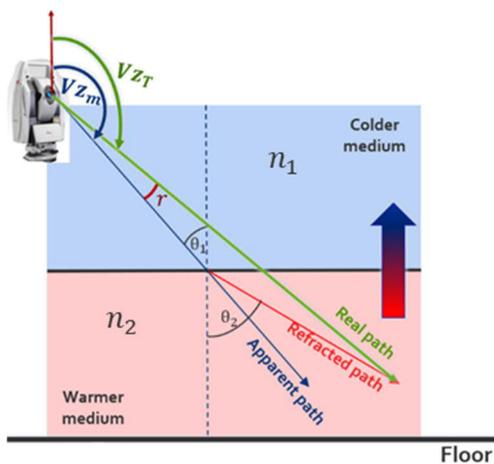


Figure 1. The effect of the refraction $n_1 > n_2$

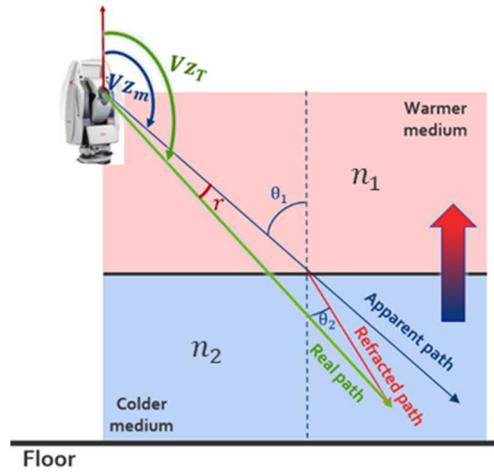


Figure 2. The effect of the refraction $n_1 < n_2$

B. The Temperature Gradient calculation

From the international literature, the equations that describe the relation of the air temperature measurements with respect to the height from the floor are the following (Kharagani 1987):

Kukkamaki's function introduced in 1938:

$$t = a + b \cdot h^c \quad (3)$$

Hugershoff's function introduced in 1907:

$$t = a + b \cdot h^2 \quad (4)$$

Reissmann's 1 function introduced in 1954:

$$t = a + b \cdot h + c \cdot h^2 \quad (5)$$

Reissmann's 2 function introduced in 1954:

$$t = a + b \cdot h + c \cdot h^2 + f \cdot h^3 \quad (6)$$

Reissmann's 3 function introduced in 1954:

$$t = a + b \cdot h + c \cdot h^2 + f \cdot h^3 + g \cdot h^4 \quad (7)$$

Heer's function introduced in 1984:

$$t = a + b \cdot \exp(c \cdot h) \quad (8)$$

Kharagani's function introduced in 1987:

$$t = a \cdot h + b \cdot h^c \quad (9)$$

Linear:

$$t = a \cdot h + b \quad (10)$$

The first derivative of each one of the above equations gives the temperature gradient. The equations below are the first derivatives of the above temperature functions.

Kukkamaki:

$$dt/dh = b \cdot c \cdot h^{-1+c} \quad (11)$$

Hugershoff:

$$dt/dh = 2 \cdot b \cdot h \quad (12)$$

Reissmann 1:

$$dt/dh = b + 2 \cdot c \cdot h \quad (13)$$

Reissmann 2:

$$dt/dh = b + 2 \cdot c \cdot h + 3 \cdot f \cdot h^2 \quad (14)$$

Reissmann 3:

$$dt/dh = b + 2 \cdot c \cdot h + 3 \cdot f \cdot h^2 + 4 \cdot g \cdot h^3 \quad (15)$$

Heer:

$$dt/dh = b \cdot c \cdot e^{c \cdot h} \quad (16)$$

Kharagani:

$$dt/dh = a + b \cdot c \cdot h^{-1+c} \quad (17)$$

Linear:

$$dt/dh = b \quad (18)$$

The main question that arises in this point is which of these functions should be used in order to calculate the appropriate temperature gradient for the measurements environment and consequently to calculate the correct refraction coefficient. It is obvious that each one of these equations gives a different value for the temperature gradient.

The most appropriate function is the one that fits best to our data. Namely, after the temperature data fitting, the one, which has R – Square closest to one, is the best.

For the selected function the calculated parameters from the data fitting should be used in the first derivate of the function in order to calculate the temperature gradient.

After these steps, the refraction coefficient can be calculated and the vertical angle correction due to refraction can be determined.

C. Refraction Coefficient k Calculation

The relation between the temperature gradient and the refractive index gradient is performed by the following equation. This formula is coming out from the simplified formula for the vertical gradient of the refractive index, which has been adopted in 1960 by the International Association of Geodesy (Kharagani 1987).

$$\frac{dn}{dz} = -\frac{78.83 \cdot 10^{-6} \cdot P}{T^2} \cdot \left(0.0342 + \frac{dt}{dz}\right) \quad (19)$$

where $\frac{dn}{dz}$ = Refractive index gradient (m^{-1})

z = Height above the ground surface

$\frac{dt}{dh}$ = Temperature gradient ($K \cdot m^{-1}$)

P = Atmospheric pressure (hPa)

T = Temperature (K)

After assumptions and calculations which are described in (Kharagani 1987) it is possible to determine the refraction coefficient k with the equation (20).

$$k = \frac{502.7 \cdot P \cdot (0.0342 + \frac{dt}{dh})}{T^2} \quad (20)$$

So, it is obvious that is crucial to determine the temperature gradient in order to calculate the refraction coefficient k .

D. Correction in vertical angles

So, the correction in the vertical angle (in rad) due to refraction can be calculated by the following equation.

$$V_{rz} = \frac{D \cdot k}{2 \cdot R} \quad (21)$$

where D = Slope distance (m)

k = Refraction coefficient

R = Mean radius of Earth (m)

V_{rz} = Correction in vertical angle due to refraction (rad)

In addition, the correction due to the earth curvature should also be considered. This correction is always subtractive, since due to the curvature of the Earth, the targets seem to be lower than they actually are. The correction of the vertical angle (in rad) due to the curvature of the Earth is calculated from the following equation.

$$V_{cz} = \sin\left(\frac{D}{2 \cdot R}\right) \quad (22)$$

where V_{cz} = Correction in the zenith angle due to earth curvature (rad)

Therefore, the final value of the vertical angle after both corrections is given by the following equation.

$$V_z = V_{zm} - V_{cz} \pm V_{rz} \quad (23)$$

where V_z = Corrected zenith angle

V_{zm} = Measured zenith angle

The sign of the refraction angle correction depends on the path in which the optical ray travels from the instrument to the target as it has been described in Figures 1 and 2. Also in all the following calculations the correction due to the Earth curvature has taken into consideration.

III. THE TT1 TUNNEL

A. Field Test – TT1 Tunnel

The Transfer Tunnel One (TT1) (Figure 3) was used until 1984 to transfer the particle beam from the Proton Synchrotron (PS) accelerator to the Intersecting Storage Rings (ISR) until it was decommissioned. Since then, this tunnel has been a field for both geodetic and other experimental applications (Boerez et al., 2012).

In the TT1 tunnel for the geodetic applications, there are established 17 permanent tripods almost placed in a line (Figure 4).

The tripods are screwed on the floor of TT1 tunnel and they have special heads in which the geodetic

instruments and the Taylor & Hobson balls can be easily placed on them (Figure 5).



Figure 3. TT1 Tunnel at CERN



Figure 4. Tripods at TT1



Figure 5. Special heads of the tripods

Moreover, along the tunnel and parallel to the permanently mounted tripods, 7 HLSystems are installed (Figure 6). Also, these systems have special heads in which the Taylor & Hubson balls can be placed.

HLSystems are located approximately every 23 m, while the tripods are spaced from 5 m to 10 m.

So, the TT1 tunnel is appropriate for the application of the TG method as it has significant equipment, easy accessibility during all year, the Hydrostatic Levelling Systems allows the comparison of the results and additional evaluation of the method.

B. The Hydrostatic Levelling Systems

The hydrostatic levelling systems are based on the principle of the communicating vessels in order to measure height differences.

In the field of particle accelerators, the use of a closed HLSystem is inevitable, as some microns, accuracy is demanded. A closed HLSystem has a fluid and air connection between all vessels of the system and no connection to the surrounding environment. An HLSystem is designed to measure local ground motions, which might affect the alignment of any accelerator installed on the same surface (Herty et al., 2004).



Figure 6. HLSystem

In TT1 tunnel at Cern there are 7 sensors of a classic HLSystem. The sensors in this system are integrated in the top of the vessel as well as electronic components for the signal processing. Capacitive measurements determine the distance to the free water surface and additional temperature measurements take place in the vessel. The accuracy of these systems is about $\pm 5 \mu\text{m}$. (Capacitive technology). The main error source of the HLSystems are the Earth's tides.

In Figure 7, the readings of one HLSystem sensor during one day in TT1 tunnel are illustrated.

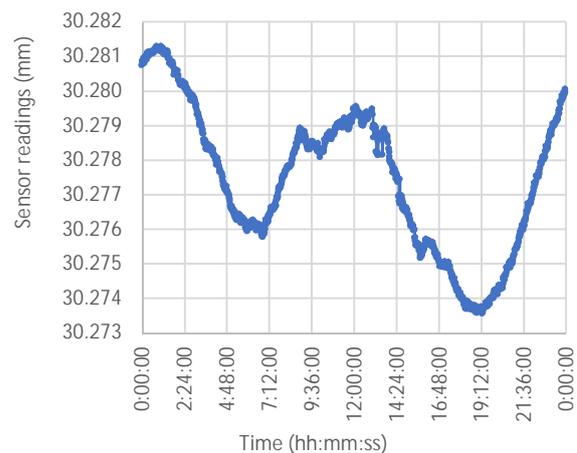


Figure 7. HLS Sensor readings during one day

IV. APPLICATION

A. Temperature Measurements

In order to apply the TG method, the temperature measurements are necessary. The EN/SMM/ESA section affords six CelsiPick temperature sensors. These temperature sensors can measure the air temperature with the desirable time interval, their size is small and their establishment very easy.

The resolution of the sensors is $0.1 \text{ }^\circ\text{C}$ and their accuracy is $\pm 0.5 \text{ }^\circ\text{C}$. The working range is from $-39 \text{ }^\circ\text{C}$ up to $+120 \text{ }^\circ\text{C}$. Before the use of the sensors a test of their performance is carried out in a certified climate – controlled room in order to calculate any differences of the nominal readings.

After the test, the 6 temperatures sensors have been established on a 3 m rod, in different heights from the

floor (0.20 m, 0.70 m, 1.20 m, 1.70 m, 2.30, και 2.90 m) (Figure 8).

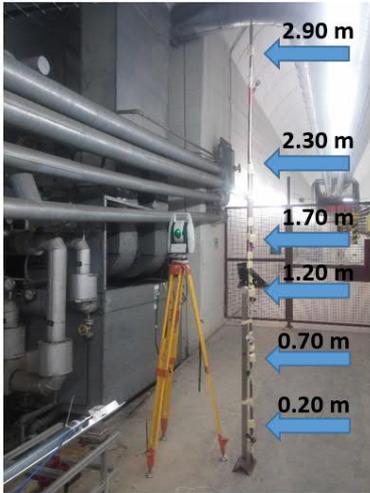


Figure 8. Distribution of the temperature sensors on the rod

This set up of the instrumentation has the advantage of being flexible due to its weight and can easily be placed next to the laser tracker in order to record the temperature data during the measurements.

In each laser tracker station, the rod with the sensors has been placed near to the instrument during the measurements which lasted approximately 50 minutes per laser tracker station. The interval between each temperature measurement was 16 seconds.

For each one of the seven stations and for each sensor (at different height from the floor) the average values of the temperatures have been calculated. The difference between the maximum and the minimum value of the average values is 3.7 °C.

In order to have a graphical representation of the temperature changes along the TT1 tunnel these data have been used in a linear interpolation in Matlab. It is obvious that at the entrance of the test area and near to the floor of the tunnel the lowest temperature has been observed. On the contrary, the highest temperatures are observed at the end of the test area and near to the tunnel ceiling. In Figure 9, the entrance of the test area is on the left side and the end is on the right side.

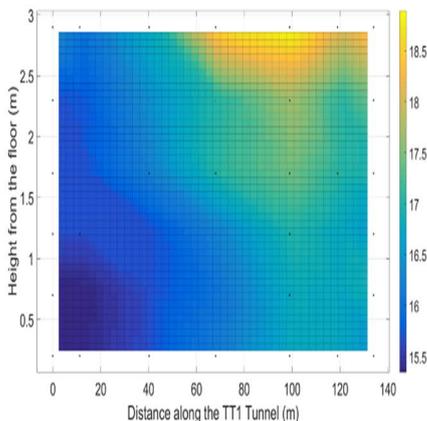


Figure 9. Temperature changes along the TT1 Tunnel

B. Geodetic Measurements

The Leica AT401 Laser Tracker (serial number 391055, firmware version 2.4.0.5494) was used for the measurements which provide angular accuracy ± 1.5 cc. (Product Brochure – Leica Absolute Tracker AT401 2018).

All the measurements towards to the points (Tripods and HLSystems) have been done only with one Taylor & Hobson ball with 1.5 inch prism inside (serial number 10782) in order to avoid eccentricity errors between the different balls. The centring of the prism inside the ball has been controlled in the last EYETS 2017/2018.

In addition, the laser tracker has been warmed up for 2.5 hours before the start of the measurements and all the geodetic equipment has been placed in TT1 Tunnel one day before the measurements in order to ensure the temperature stability.

The no commercial software Carnet4000 (version 2.05.05, developed by the personnel of EN/SMM group at CERN) has been used for the field measurements.

In Figure 10, the seven instrument stations are marked in red, the position of the 17 tripods are marked in blue and the position of the 7 HLSystems are marked in green.

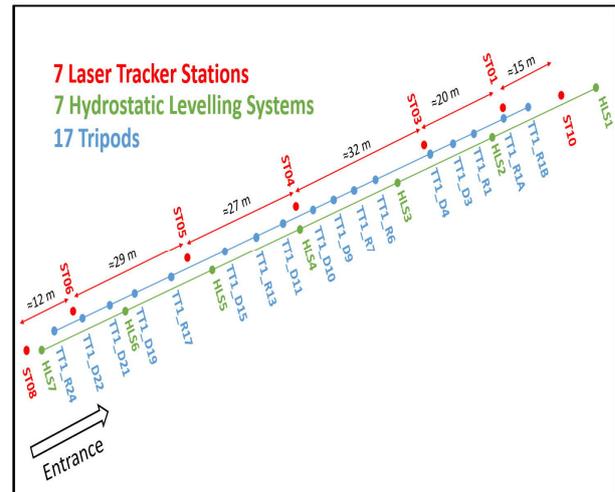


Figure 10. The point distribution in TT1 Tunnel

C. Calculations

Therefore, two different 3D network adjustments have been done using LGC (Logiciel Général de Compensation) software (No commercial software).

The LGC software, which is also created by the EN/SMM group personnel, gives the opportunity to the user to compensate 3D network adjustments of multiple natures. In this research work the 3D network adjustments have been done using the key work *SPHE. When the user compensates the 3D network adjustment with this key word, then the zenith angles are corrected directly due to earth curvature.

This 3D network has been adjusted two times with different input data concerning the zenith angles:

- in the first time, the measured zenith angles have been corrected only by the earth curvature.
- in the second time, the 3D network adjustment has been done with the corrected zenith angles due to earth curvature and due to refraction using the temperature gradient method.

The minimum constrains that have been considered for the adjustments are one point fixed and one direction fixed.

For the mathematical model of the adjustment have been used, 126 observations of horizontal angles, 126 observations of zenith angles and 252 observations of distances. The number of the unknowns was 97.

For the correction of the zenith angles due to refraction with the temperature gradient method, the refraction coefficient per laser tracker station has been determined. These values of the refraction coefficients are illustrated in the next table. For these calculations the temperature function Reissmann 3 has been selected due to the fact that this function is the one that fits best to our temperature data.

Table 1. The values of coefficient k per station

Station	k
ST01	1.48
ST03	2.76
ST04	2.66
ST05	3.67
ST06	1.55
ST08	3.90
ST10	1.08

The corrections of zenith angles due to refraction for this 3D network have been ranged from 0.1 cc to 27.7 cc. The corresponding corrections due to the earth curvature have been ranged from 0.1 cc to 6.5 cc.

V. RESULTS

A. Residuals analysis of the 3D network adjustment

So, for the first type of adjustment of each 3D network the zenith angles have been corrected only due to the curvature of earth. For the second type of adjustment of each 3D network the zenith angles have been corrected due to curvature of earth and due to refraction.

In the following figure the average of the zenith angle residuals per adjustment are illustrated. For the 3D network adjustments in which the zenith angles have been corrected due to refraction (TG method), it is obvious from the Figure 11 (orange bars) that the average of the residuals is getting closer to zero (below 0.5 cc). This fact means that the systematic error of the refraction has been reduced significantly.

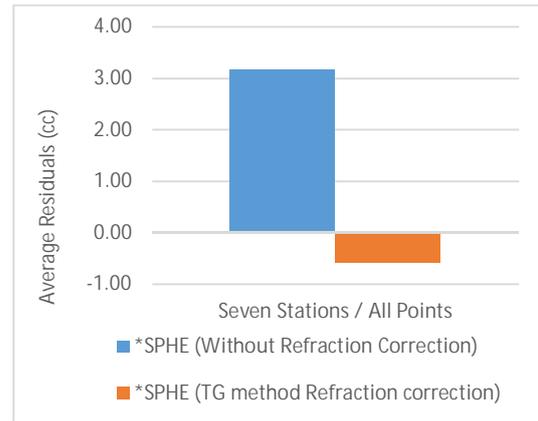


Figure 11. Average of zenith angle residuals per 3D adjustment

In Figure 12, the standard deviation of the zenith angle residuals per adjustment for the 3D network are illustrated. For the 3D network adjustments in which the zenith angles have been corrected due to refraction (orange bars), it is obvious that the standard deviation of the residuals is getting closer to the instrument specification for the angle measurements (± 1.5 cc).

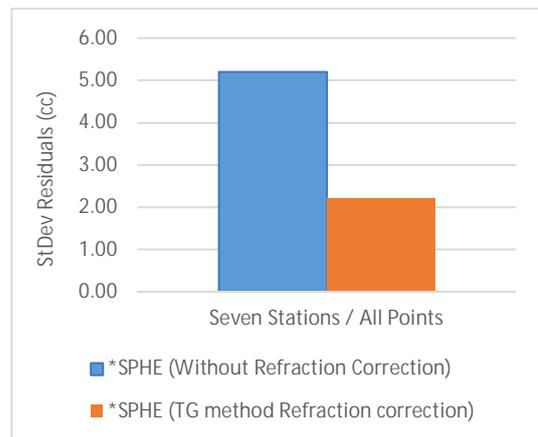


Figure 12. Standard deviation of zenith angle residuals per 3D adjustment

B. Height difference comparisons with the HLSystems

The existence of the HLS points gives the opportunity to compare the nominal height differences between the 7 HLS points with the height differences which are calculated from the Z – coordinates after the 3D network adjustments.

In Figure 13, the orange bars represent the differences that are come out after the 3D network adjustment with the corrected zenith angles with the TG method and in blue bars are illustrated the differences that come out after the 3D adjustment with the zenith angles which have corrected only by the curvature of earth. In Figure 13, all the height differences significantly approaches the nominal values after the correction in the zenith angles due to refraction with the TG method.

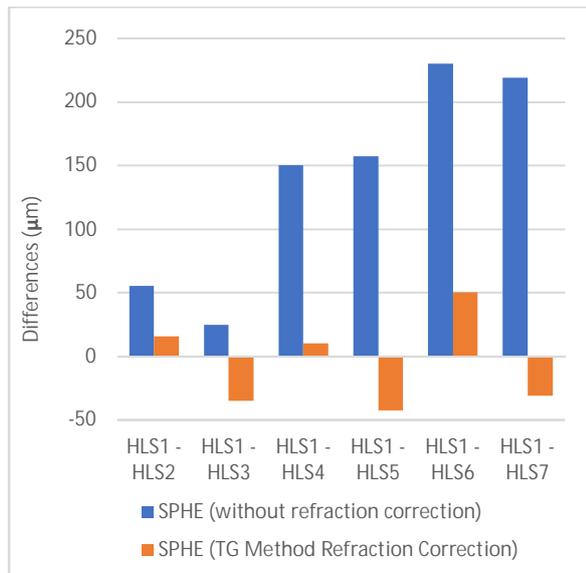


Figure 13. Comparison of the height difference of the HLSystems with the nominal height differences

In Table 2 the maximum differences of the comparison of the nominal height differences between the seven HLSystems with the height differences, which are calculated from the Z – coordinates are marked in red color.

The maximum difference from the nominal values is 230 µm before the refraction corrections in the vertical angles and is reduced to 50 µm after the correction with the TG method.

Table 2. Differences of the height differences of the HLSystems with the nominal height differences

Height Difference	*SPHE without Refraction Correction (µm)	*SPHE TG Refraction Correction method (µm)
HLS1 – HLS2	55	15
HLS1 – HLS3	25	-35
HLS1 – HLS4	150	10
HLS1 – HLS5	157	-43
HLS1 – HLS6	230	50
HLS1 – HLS7	219	-31

VI. CONCLUSIONS

The aim of this research work is to provide the correction of the vertical angles due to refraction in geodetic measurements. A new method by means of the temperature gradient calculation is applied for this purpose.

According to this method, the air temperature measurements have been registered at each instrument station during the measurement of vertical angles. So, the temperature gradient at this place and the refraction coefficient k can be calculated.

Eight different temperature functions have been tested in order to fit the data of the temperature measurements. The Reissmann 3 function proved to be the most appropriate for these data. Using this

function, the refraction coefficients in each instrument station have been calculated and the vertical angles have been corrected due to refraction.

In order to emerge the significance of the refraction correction two separate adjustments with and without these corrections are carried out.

After the adjustments, the analysis of the results a significant reduction of the averages of the residuals from 3 cc to 0.5 cc is registered when the refraction corrections have been applied. This proves that the systematic error of the vertical angles due to the effect of refraction is limited by 80%.

Also, a significant reduction almost 70% is registered for the residuals of the vertical angles when the refraction corrections have been applied. This value is very close to the nominal accuracy of the laser tracker (± 1.5 cc).

Also, the differences between the nominal height differences of the HLSystems and the calculated differences by the 3D network fluctuate between -43 µm to 50 µm. This fact certifies the correctness of the TG method.

As the results are very promising, the proposed methodology gives an alternative against the geodetic levelling, especially for internal high accuracy applications as in tunnels.

In addition, a proposal for the improvement of the instrumentation is the replacement of the temperature sensors with a Distributed Temperature Sensing system. These are optoelectronic devices, which measure temperatures by means of optical fibres functioning as linear sensors. In this case, temperatures are recorded along the optical sensor cable, thus not at points, but as a continuous profile. Using these systems, the profile of the temperature gradient will be more accurate as well as the determination of the refraction coefficient k .

The next challenge is to test the method to external networks, where the air temperature measurement conditions are more difficult and inaccurate in order to improve them.

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