Deriving Orthometric Heights from GPS Measurements Using a Height Reference Surface

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Key words: Geiod, Deflection of the Vertical, Heights.

ABSTRACT

The use of navigation satellites for geodetic surveying is common practice today. The combined use of satellite receivers with total stations and levels is also relatively frequent.

The heights, or height differences, derived from satellite measurements refer to an ellipsoid. Orthometric heights refer to a geoid. To derive orthometric heights from satellite measurements and to combine satellite with traditional measurements, both require knowledge of the relationship between geoid and the ellipsoid in the area.

This paper describes a simple way of deriving orthometric heights from satellite measurements in areas where the geoid varies significantly. By use of a height reference surface the deflections of the vertical are computed and height differences are corrected.

The method is tested on a few projects with GPS data. The method seems to work well also in areas where the deflection of the vertical vary considerably. The method also gives good results when the height reference surface and the computations are referred to different geodetic reference frames. In addition a method to visualize the variation of the deflection of the vertical by computing the curvature of the height reference surface is shown.

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1 HEIGHT REFERENCE SURFACE

The geoid is essential when high precision height determinations are obtained with GPS. The basic equation is

$$H = h - N \qquad (1)$$

where H is the orthometric height, h is the ellipsoidal height and N is the geoid height.

If normal heights are used instead of orthometric heights, an equivalent equation can be expressed:

$$H^* = h - \zeta \qquad (2)$$

where H* is the normal height and ζ is the quasigeoid height. The difference between the geoid and the quasigeoid is dependent on the Bouguer gravity anomaly and the height. This difference locally mirrors the topography, and typically is some cm [Forsberg, 2000].

When practical GPS height determination is done, the following relation is often interesting:

$$H_{IMPLEM} = h_{GPS} - N_{REF} \qquad (3)$$

H_{IMPLEM} is the height referred to the implemented height system. h_{GPS} is the ellipsoidal height above the used ellipsoid. N_{REF} will then become the difference between these two heights, and are named height reference surface.

Both the quasigeoid and the height reference surface are often called “geoid”.

The distinction between the height reference surface and the «theoretical» geoid (or quasigeoid), is the error or constraint in the applied height system (H_{IMPLEM}).

Because of geodynamic effects, especially land uplift, every quantity in the equation (3) has to be referred to the same epoch.

2 THE METHOD

Basically the method is to correct the GPS measured ellipsoidal heights to orthometric height measurements. Consider figure 1.

Ellipsoidal height difference:

$$dh = h_A - h_B$$

Orthometric height difference:

$$dH = H_A - H_B$$

Relation between dh and dH:

$$dh = h_A - h_B = H_A + N_A - H_B - N_B$$

$$dh = dH + dN$$

$$dH = dh - dN \qquad (4)$$

where $dN = N_A - N_B$ is the geoid height difference between the two points. Equation (4) describe the relation between ellipsoidal and orthometric height difference expressed by geoid heights.

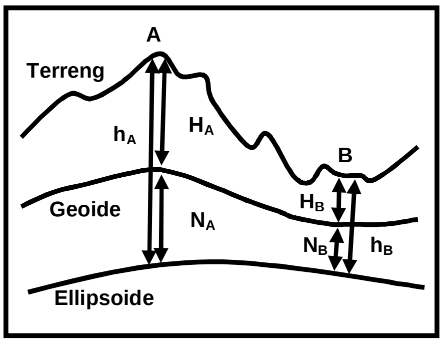

Figure 1. Relation between orthometric, ellipsoidal and geoid heights.

Example:
Point B has a given orthometric height (H_B=100m). The observed baseline between the two points is measured with use of GPS ($dh=h_A-h_B$=50m). A geoid model exists and the geoid heights on the points can be found ($dN=N_A-N_B$=10cm). The orthometric height at point A (H_A) can be expressed as:

$$H_A = H_B + dh - dN$$
$$H_A = 100m + 50m - 0.1m = 149.9m$$

The quantities in equation (4) are more or less known, and can be expressed by their standard deviations.

$$\sigma_{dH}^2 = \sigma_{dh}^2 + \sigma_{dN}^2 \qquad (5)$$

3 IMPLEMENTATION OF THE METHOD

The method is implemented in an existing surveying software package, V/G-Land, developed by Norkart AS. This is done to empirically examine the method. This software executes the horizontal adjustment in the map projection plane and the vertical adjustment with the geoid as reference. GPS observations are transformed from the geocentric system to the local topocentric coordinate system. A GPS baseline will then consist of a slope distance, azimuth and a zenith distance (angle) or height difference that is related to the ellipsoid. Since the vertical adjustments are done relating to the geoid, the ellipsoidal height observation has to be corrected for the deflection of the vertical. As mentioned earlier this can be achieved by using geoid heights at the points.

The Norwegian Map Authority supports Norwegian geodesists and surveyors with two reference surfaces, HREF2000 and VREF1996, by means of a DLL interface. These are based on a geoid and a quasigeoid. Functionality to get the geoid height from this component is made.

The geoid heights are considered as corrections and not as observations. And so equation (5) is simplified to:

$$\sigma_{dH}^{2} = \sigma_{dh}^{2}$$

Thus σ_{dH} is smaller than it should be. On the other hand it is hard to tell how big σ_{dN} really should be. σ_{dN} does not express an absolute standard deviation of the geoid height, but deviation of differences of geoid heights. This differentiate standard deviation is undoubtedly dependent on the spatial length of the GPS baseline. In addition, it will be dependent on horizontal variations in the reference surface model accuracy.

4 SOME TESTS

Some tests are carried out on data given of the Norwegian Map Authority (Statens Kartverk), The Norwegian Road Authority (Vegvesenet), a Norwegian municipality (Ski kommune) and a Norwegian surveying company (GEFO). All projects are measured only with GPS. Blunder-detection is done and it is assumed that no observations contain blunders. All GPS baselines are transformed to the map projection. Weights are taken from baseline processing. Adjustments are carried out only with the height component of the observations. The estimated standard deviation of unit weight σ_0 is used as a measure of the global accuracy of the height determination. Computing a confidence interval around the estimated blunder forms internal reliability. The impact on deflection of the vertical (external reliability) caused by this maximal blunder, is estimated when the deflection of the vertical is estimated.

V/G-Land version 10.2 compiled 20.03.2000 and V/G-Trans 2.0 compiled 16.03.2000 are the two software packages used. Both reference surfaces from the Norwegian Map Authority are used.

- VREF1996: skvref96.bin, 1998-06-04.
  This reference surface is based on the NKG96n geoid, and is the official reference surface provided by the Norwegian Map Authority.
- HREF2000: href2000b.bin, 2000-10-05.
  This reference surface is based on a quasigeoid. It is not an official reference surface, but the Norwegian Map Authority has kindly lent it out.

A total of 36 adjustments are carried out on four projects. Two of the projects are adjusted in two different reference frames. Some of the results from the computations are shown in table

1, section 8. The four projects are described below.

4.1 SKAUR

This project named SKAUR is handed over from the Norwegian Map Authority. The area extends 60 km east west and 50 km north south, and covers a mountainous region. The geoid in this area is significantly uneven. Therefore it is not possible to model the area with constant deflection of the vertical with success. The project consists of 232 GPS baselines, 22 fixed heights, and 57 unknown heights. It is assumed that both observations and fixed points have high quality.

In Norway, municipalities use a reference frame called NGO1948. This is a national reference frame that is defined with zero geoid height and zero deflection of the vertical in Oslo, Norway's capital. Another official reference frame is EUREF89. This is ITRF at 1. January 1989, and thus has typically 40 meter geoid height. These two reference frames use different ellipsoids.

The SKAUR project is computed in both NGO1948 and EUREF89. Both reference surfaces VREF1996 and HREF2000 are used to correct for deflection of the vertical. Computations without correcting are also tested. Adjustments with and without a constant deflection of the vertical as a parameter are done. This makes a total of (2 coord.syst · 3 corr.types · 2 parameter types =) 12 combinations.

4.2 GEFOAUR

This project was designed with the purpose of measuring control points for photogrammetry. The project area is about 25 km in diameter and is partly the same as in SKAUR. This project is computed in EUREF89.

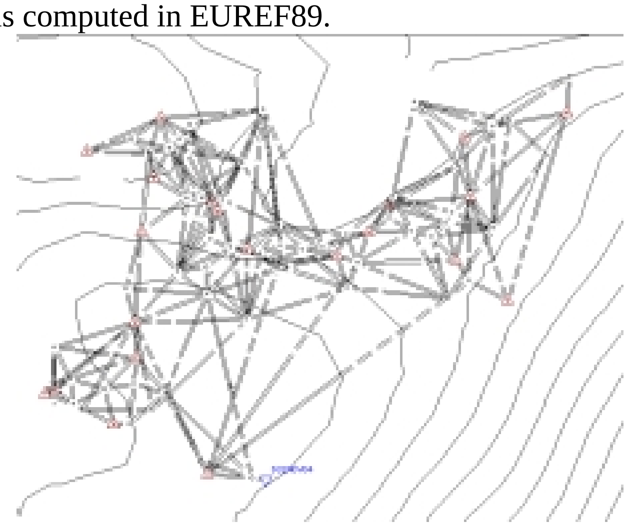

Figure 2 (left): Project SKAUR. Geoid height is drawn with 20 cm contour interval.

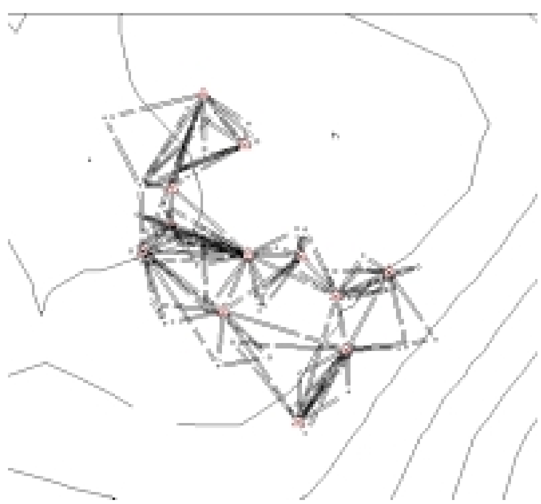

Figure 3 (right): Project GEFOAUR. Geoid height is drawn with 20 cm contour interval.

4.3 SKI

Ski municipality had a GPS campaign to re-measure their main net. The project extends is about 23 km north-south and 16 km east-west. The deflection of the vertical is almost constant in this area. Adjustments are computed in both the NGO1948 and EUREF89 reference frames.

4.4 VDSULE

This project is organized and measured by the Norwegian Road administration. The project area is narrow and over 24 km long. It is measured using the GPS RTK technique and the accuracy is expected to be slightly worse than the two previously described projects. The 29 fixed points are measured by levelling. The project is only computed in NGO1948.

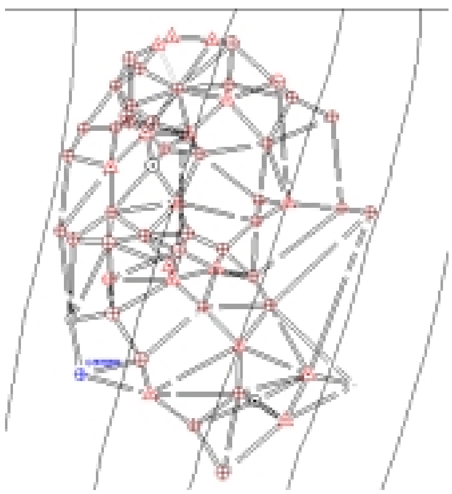

Figure 4 (left): Project SKI. Geoid height is drawn with 20 cm contour interval.

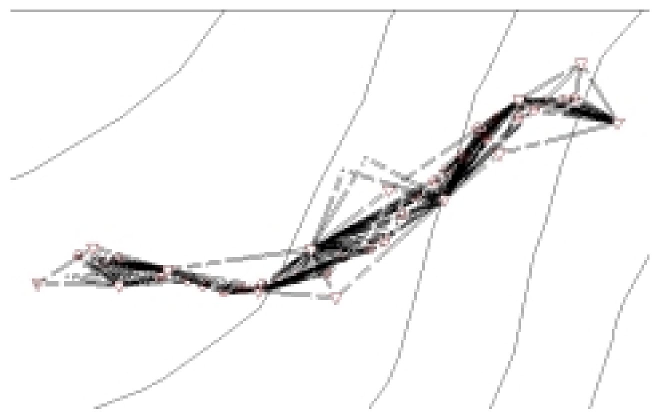

Figure 5 (right): Project VDSULE. Geoid height is drawn with 20 cm contour interval.

5 RESULTS OF THE TESTS

A selection of results are attached in table 1, section 8, together with a list of the abbreviations used. It would be an advantage to add more observations because it is hard to draw conclusions with only four tests, and the conclusions have to be seen from that point of view.

To decide if one adjustment has less constraint than another, their variances of unit weight are compared. The following hypotheses are formed:

$$H_0: \quad S_1^2 / S_2^2 \leq 1$$
$$H_1: \quad S_1^2 / S_2^2 > 1$$

where S_1 and S_2 are the estimated standard deviations of unit weight. It can be shown that S_1^2 / S_2^2 follows a Fisher distribution [Høyland, 1986], and that conclusion could be drawn by $S_1^2 / S_2^2 \geq F_{\alpha,f1,f2}$. This test assumes the two observations to be independent. In this case they are not. For this reason the probability of rejecting the H_0 hyposesis is less, and the test strength is weakened. The significance level is chosen to be 5% for all tests that are carried out.

5.1 The Method's Ability to Reduce Constraint in the Adjustment Model

The first tests are done in EUREF89, the same reference frame as the reference surface. Two types of adjustments are done. The first adjustment is with one pair of deflections of vertical as unknown parameters. The second is with correcting the observations using a reference surface. A comparison between these two adjustments (table 2) shows that the adjustment constraint is less when correcting the observations for the reference surface compared to adjustments where the geoid is modelled with a constant deflection of the vertical. The method seems to work well in these cases. The constraint in SKI, meanwhile, is not reduced significant, probably because the geoid in this area approximately a plane.

		Constant defl. of vert				Geoid is VREF1996				Test	
Project	Datum	Corr	DOV	f	S0	Corr	DOV	f	S0	Ftest	Ftab
SKAUR	EUREF	No	x	173	14.9	VREF		175	3.35	**4.46**	1.28
GEFOAUR	EUREF	No	x	94	5.46	VREF		96	2.03	**2.70**	1.40
SKI	EUREF	No	X	113	1.07	VREF		115	0.92	1.18	1.36

Table 2: Testing if estimated standard deviation on unit weight is less when correcting observations for deflection of the vertical with a reference surface. In the case without correction, the deflection of the vertical is an unknown parameter in the adjustment.

5.2 Simultaneously Correcting and Estimating Deflection of the Vertical

In addition to correct the observations for the deflection of the vertical, a constant pair of parameters modelling the deflection of the vertical are estimated in the adjustment. We expect the constraint to some extent to be reduced since the number of parameters is increasing. The estimated parameters of deflection of the vertical have to be tested for significance unequally to zero. Both the deflection of the vertical and its standard deviation are estimated. The test can be performed using a student-T distribution.

For the projects SKAUR and VDSULE a significant deflection of the vertical was estimated even though all observations are corrected for it. This is perhaps unexpected and the quality of the reference surface was questioned. As shown in the previous section, the method is a good working model and reduces the strain. The significant deflection of the vertical indicates a tilt, or a remaining signal with long wavelength, in the reference surface.

The reliability of the estimated deflection of the vertical is computed as the impact of one maximum undetectable blunder on the estimated deflection of the vertical. Comparing the reliability with the estimated value is one way to evaluate the reliability of deflection of the vertical. The question is what limit should be defined as an unreliable estimate of deflection of the vertical. In this test an estimated value is reliable if it is smaller than 10 times the external reliability. The external reliability of deflection of the vertical varies between the projects. The reliability of deflection of the vertical is poor in SKI, while in the mountainous area it is a bit better, but still not really good. Even if the estimated deflection of the vertical is significant, it is not estimated to be reliable.

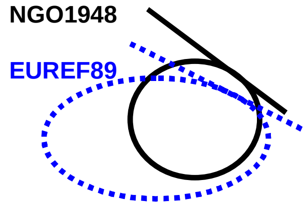

Figure 6: Two different ellipsoids

5.3 Adjustment and Corrections in Two Different Reference Frames

The available reference surfaces VREF1996 and HREF2000 use the EUREF89 ellipsoid. For those projects that are adjusted in NGO1948 these available reference surfaces is used. EUREF89 and NGO1948 are not coincident because of differences in size, shape, orientation and position of the ellipsoids. The differences are shown in figure 6. These differences can be modeled in the adjustment by estimating some extra parameters. Estimating the scale compensates the error caused by wrong geoid height since geoid height is correcting the distance part of the observation in proportion to the baseline length. Difference in tilt on the local ellipsoid normal is compensated by estimating a constant deflection of the vertical for the projects. In this case the estimated “deflection of the vertical” will handle both the effect of deflection of the vertical and the difference between the two ellipsoids. This is the reason for the large estimated deflection of the vertical.

The adjustment in SKI is one example that illustrates this fact. Adjustments with and without correcting the deflection of the vertical show that the estimated deflection of the vertical is nearly six times larger with correction using a height reference surface (η: 0.88mgon>0.15mgon). Table 3 shows that the method works well for the project SKAUR. As in the adjustments in EUREF89, the largest gain is where the deflection of the vertical varies, i. e. mountainous areas. Figures 4 and 5 show the geoid in SKI and VDSULE which appear to be approximately a plane. If such is the case, it will explain that the method does not have a significant effect on these two projects.

		No Correction				Correction using VREF1996				Test	
Project	Datum	Corr	DOV	f	S0	Corr	DOV	F	S0	Ftest	Ftab
SKAUR	NGO	No	x	173	14.91	VREF	x	173	3.172	**4.70**	1.28
VDSULE	NGO	No	x	731	0.200	VREF	x	731	0.177	1.13	1.40
SKI	NGO	No	x	108	1.076	VREF	x	108	0.907	1.19	1.36

Table 3: Test of estimated standard deviation of unit weight with NGO1948 is less with correction than without correction.

5.4 HREF2000 and VREF1996

The major differences between the two reference surfaces can be summarised bysaying that HREF2000 is based on a quasigeoid, is newer, and more data is used in geoid determination. The question is whether it possible to tell which of them is the best. Roughly speaking, Table 4 shows that HREF2000 is better than VREF1996. The reduction of constraint is however so small that it is not significant. One exception, on the other hand, is in the project GEFOAUR in NGO1948 where HREF is significant better than VREF1996.

A radical advance in development of more accurate reference surfaces is to be expected. More accurate global geoid models will emerge through satellite missions like CHAMP, GRACE and GOCE (Pettersen, Solheim 2000). Larger access to gravity data, better terrain models and more GPS-levelling will improve the regional geoid fitting (Forsberg, 2000).

The magnitude of land uplift is for some areas in Norway 1 cm/year. Keeping this fact in mind together with the development of new and more accurate reference surfaces, it is natural to use the most up-to-date reference surface data.

Project	Datum	Corr	DOV	f	S0	Corr	DOV	f	S0	Ftest	Ftab
SKAUR	EUREF	Vref		175	3.3538	Href		175	2.7117	1.237	1.283
SKAUR	EUREF	Vref	x	173	3.2215	Href	x	173	2.529	1.274	1.285
SKAUR	NGO	Vref		175	21.8598	Href		175	21.9074	0.998	1.283
SKAUR	NGO	Vref	x	173	3.1727	Href	x	173	2.5211	1.258	1.285
VDSULE	NGO	Vref		733	0.9075	Href		733	0.9114	0.996	1.129
VDSULE	NGO	Vref	x	731	0.1773	Href	x	731	0.1759	1.008	1.129
SKI	EUREF	Vref		115	0.9153	Href		115	0.8961	1.021	1.361
SKI	EUREF	Vref	x	113	0.9072	Href	x	113	0.8948	1.014	1.364
SKI	NGO	Vref		110	7.7355	Href		110	7.7252	1.001	1.37
SKI	NGO	Vref	x	108	0.9503	Href	x	108	0.9336	1.018	1.374
GEFOAUR	EUREF	Vref		96	2.025	Href		96	1.6816	1.204	1.401
GEFOAUR	EUREF	Vref	x	94	1.6309	Href	x	94	1.1226	**1.453**	**1.406**

Table 4: Test if the reference surface VREF1996 is better than HREF2000.

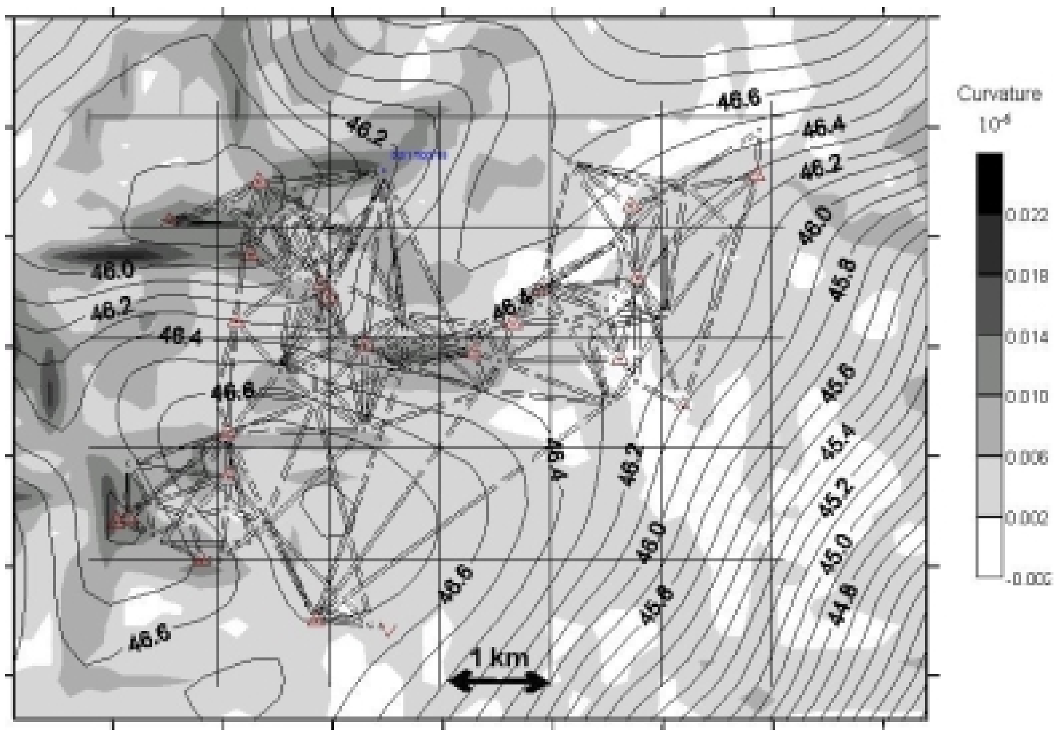

Figure 8: Large curvature, dark color. The geoid is plotted with a contour interval at 10 cm. This area is identical with the SKAUR project area.

6 IS THE GEOID SMOOTH?

The test result shows that the gain of using this method is not always significant. If the project area has an approximately constant deflection of the vertical, the method is of course not suitable. A natural question is when to assume a constant deflection of the vertical and when to model or correct for the geoid in a more sophisticated fashion. There are a lot of criteria to determine the answer to this question. One method could be studying residuals after fitting a model with constant deflection of the vertical. This can be reached locally by plotting geoid heights (corrected for curvature of the earth's surface) on a map, where a linear regression fit is carried out. This will probably work well, but it requires knowledge of which points the project consists of.

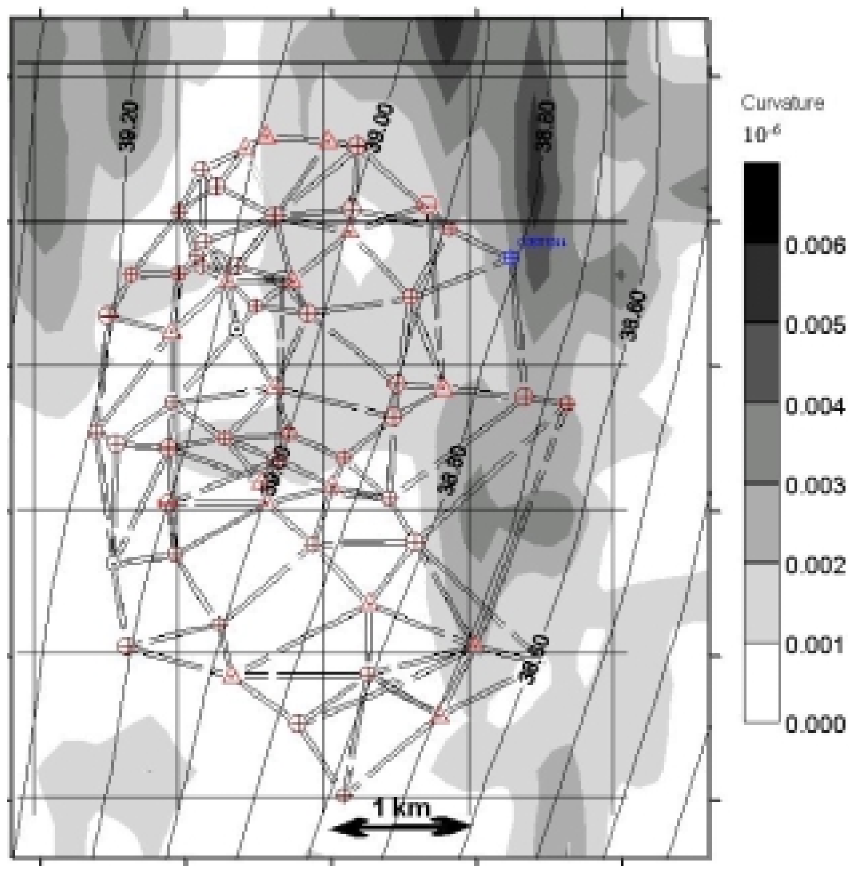

Figure 9: Large curvature, dark color. The geoid is plotted with a contour interval at 10 cm. This area is identical with the SKI project area.

Another method that will be discussed and tested below, is to focus on the curvature of the geoid. The basic idea is that if the project area has modest curvature, constant deflection of the vertical (locally a plane) is sufficient. This is an a priori method that does not consider locations of any measuring points. It does follow that this method can be used to map where the deflection of the vertical varies. This can be a tool of great value to help telling before measuring where region of problems are.

Assume a surface N in a limited area. The partial derivative of this surface is the tilt of this surface. If this surface is the geoid, the tilt is the deflection of the vertical.

$$\xi = -\partial N/\partial x\ ,\ \eta = -\partial N/\partial y \qquad (6)$$

The partial derivative of deflection of the vertical gives us the surface curvature.

$$k_x = \partial^2 N/\partial x^2,\ k_y = \partial^2 N/\partial y^2 \qquad (7)$$

This method is tested in the areas covered by project SKAUR and SKI. First a 1km grid of VREF1996 was made. The secondly partial derivative of this grid was derived as two new grids. One grid is formed from the two second partial derivative grids by computing the "polar sum" for each grid-point:

$$k_{(i,j)}{}^2 = k_{x(i,j)}{}^2 + k_{y(i,j)}{}^2 \qquad (9)$$

This "surface" is plotted together with geoid heights and observations in figures 8 and 9. Dark color indicates large curvature.

It is easy to tell that the deflection of the vertical varies a lot more in figure 8, compared to figure 9. The advantage of using this method is the ability to quantify the divergence from a constant deflection of the vertical before planning the survey. That suggests that maps covering large regions may become necessarily produced.

7 CONCLUSIONS

The results of the few tests completed can be summed up as follows:

- The method works well in areas where the deflection of the vertical varies. This happens because the method is modeling the geoid.
- Simultaneous correction and estimating the deflection of the vertical may reveal errors in the reference surface.
- If the adjustment is done in a different reference frame than the reference surface referred to, constant scale and deflection of the vertical have to be simultaneously estimated.
- The method work well even when the reference surface and the adjustment do not refer to the same reference frame.

Because of geodynamic effects and the fact that reference surfaces progressively get better, it is in addition to be expected that the reference surface has to be "fresh".

8 TABLE 1

Project **SKAUR**
Datum **EUREF89 zone 32**

Corr	DOV	n	e	F	S0	DOVX	DOVY	SDOVX	SDOVY	TSLX	TSLY	Ttab	PDOVX	PDOVY	TPLX	TPLY
no		232	57	175	16.3779	x	x	X	x	x	x	x	x	x	x	x
no	x	232	59	173	14.9548	0.00074	-0.0003	0.00012	0.00014	6.167	-2.14	2.26	-0.00033	-0.0003	-2.24	1
Vref		232	57	175	3.3538	x	x	X	x	x	x	x	x	x	x	x
Vref	x	232	59	173	3.2215	0.0001	-0.00007	0.00003	0.00003	3.333	-2.33	2.26	0.00008	-0.00012	1.25	0.583
Href		232	57	175	2.7117	x	x	X	x	x	x	x	x	x	x	x
Href	x	232	59	173	2.529	0.00009	-0.00009	0.00002	0.00002	4.5	-4.5	2.26	0.00006	-0.00009	1.5	1

Project **SKAUR**
Datum **NGO1948 zone II**

Corr	DOV	n	e	f	S0	DOVX	DOVY	SDOVX	SDOVY	TSLX	TSLY	Ttab	PDOVX	PDOVY	TPLX	TPLY
no		232	57	175	28.9828	x	x	x	x	x	x	x	x	x	x	x
no	x	232	59	173	14.9085	0.00141	-0.00283	0.00012	0.00014	11.75	-20.2	2.26	-0.00032	-0.0003	-4.41	9.433
Vref		232	57	175	21.8598	x	x	x	x	x	x	x	x	x	x	x
Vref	x	232	59	173	3.1727	0.00077	-0.0026	0.00003	0.00003	25.67	-86.7	2.26	0.00008	-0.00012	9.625	21.67
Href		232	57	175	21.9074	x	x	x	x	x	x	x	x	x	x	x
Href	x	232	59	173	2.5211	0.00077	-0.00262	0.00002	0.00002	38.5	-131	2.26	0.00006	-0.00009	12.83	29.11

Project **VDSULE**
Datum **NGO1948 zone II**

Corr	DOV	n	e	F	S0	DOVX	DOVY	SDOVX	SDOVY	TSLX	TSLY	Ttab	PDOVX	PDOVY	TPLX	TPLY
no		840	107	733	0.4335	x	x	x	x	x	x	x	x	x	x	x
no	x	840	109	731	0.1998	0.00287	-0.00141	0.00006	0.00003	47.83	-47	2.25	-0.00008	-0.00003	-35.9	47
Vref		840	107	733	0.9075	x	x	x	x	x	x	x	x	x	x	x
Vref	x	840	109	731	0.1773	0.00184	-0.00287	0.00005	0.00003	36.8	-95.7	2.25	-0.00007	-0.00003	-26.3	95.67
Href		840	107	733	0.9114	x	x	x	x	x	x	x	x	x	x	x
Href	x	840	109	731	0.1759	0.00174	-0.00285	0.00005	0.00003	34.8	-95	2.25	-0.00007	-0.00003	-24.9	95

Project **SKI**
Datum **EUREF89 zone 32**

Corr	DOV	n	e	f	S0	DOVX	DOVY	SDOVX	SDOVY	TSLX	TSLY	Ttab	PDOVX	PDOVY	TPLX	TPLY
no		158	43	115	5.7505	x	x	x	x	x	x	x	x	x	x	x
no	x	158	45	113	1.0762	-0.00069	0.00201	0.00003	0.00004	-23	50.25	2.27	-0.00005	0.00008	13.8	25.13
Vref		158	43	115	0.9153	x	x	x	x	x	x	x	x	x	x	x
Vref	x	158	45	113	0.9072	-0.00004	-0.00005	0.00003	0.00003	-1.333	-1.67	2.27	-0.00004	-0.00006	1	0.833
Href		158	43	115	0.8961	x	x	x	x	x	x	x	x	x	x	x
Href	x	158	45	113	0.8948	-0.00002	-0.00004	0.00003	0.00003	-0.667	-1.33	2.27	-0.00004	-0.00006	0.5	0.667

Project **SKI**
Datum **NGO1948 zone III**

Corr	DOV	n	e	f	S0	DOVX	DOVY	SDOVX	SDOVY	TSLX	TSLY	Ttab	PDOVX	PDOVY	TPLX	TPLY
no		154	44	110	2.1777	x	x	x	x	x	x	x	x	x	x	x
no	x	154	46	108	1.099	0.00015	-0.00069	0.00004	0.00004	3.75	-17.3	2.27	-0.00005	0.00009	-3	-7.667
Vref		154	44	110	7.7355	x	x	x	x	x	x	x	x	x	x	x
Vref	x	154	46	108	0.9503	0.00087	-0.00274	0.00003	0.00003	29	-91.3	2.27	0.00004	-0.00007	21.75	39.14
Href		154	44	110	7.7252	x	x	x	x	x	x	x	x	x	x	x
Href	x	154	46	108	0.9336	0.00088	-0.00273	0.00003	0.00003	29.33	-91	2.27	-0.00004	-0.00006	-22	45.5

Project **GEFOAUR**
Datum **EUREF89 zone 32**

Corr	DOV	n	e	f	S0	DOVX	DOVY	SDOVX	SDOVY	TSLX	TSLY	Ttab	PDOVX	PDOVY	TPLX	TPLY
no		146	50	96	6.6196	x	x	x	x	x	x	x	x	x	x	x
no	x	146	52	94	5.4625	-0.00011	-0.00073	0.00011	0.00011	-1	-6.64	2.28	0.0004	-0.00043	-0.28	1.698
Vref		146	50	96	2.025	x	x	x	x	x	x	x	x	x	x	x
Vref	x	146	52	94	1.6309	-0.00006	-0.00022	0.00003	0.00003	-2	-7.33	2.28	0.00011	-0.00009	-0.55	2.444
Href		146	50	96	1.6816	x	x	x	x	x	x	x	x	x	x	x
Href	x	146	52	94	1.1226	-0.0001	-0.00021	0.00002	0.00002	-5	-10.5	2.28	0.00006	-0.00008	-1.67	2.625

8.1 Guide to the Tables

There is one table for each project for each reference frame. The two projects SKI and SKAUR are both adjusted in two reference frames: EUREF89 and NGO1948. GEFOAUR and VDSULE are adjusted in one reference frame, respectively EUREF89 and NGO1948. The rows in the table show different adjustments. Columns show conditions and results.

List of Abbreviations Used in Tables:

Project: Name of the project to which the adjustment belongs.
Datum: Adjustments are carried out in two reference frames: EUREF89 or NGO1948.

Corr: indicate if VREF1996 (Vref) or HREF2000 (Href) is used to correct the observations, or if no correction on deflection of the vertical is added. A description of these two reference surfaces can be found in sections 3 and 4.
DOV: X indicates that one constant pair of deflection of the vertical parameters is estimated.
n: Number of equations /observations.
e: Number of unknown parameters (heights and deflection of the vertical) in the adjustment.
f: Degrees of freedom / redundant observations. f = n - e.
Spvv: Estimated sum of the squared residuals.
S0: Estimated standard deviation of unit weight.
DOV: Estimated deflection of the vertical, split into north-south (X) and east-west (Y) components.
SDOV: Estimated standard deviation of deflection of the vertical.
PDOV: Estimated external reliability of deflection of the vertical.
TSL: The test statistic DOV/SDOV. Used to make a decision about whether the deflection of the vertical is significantly different from zero. Computed in north-south (X) and east-west (Y) components.
If TSL > Ttab the deflection of the vertical is significantly non-zero.
Ttab: Critical value used in hypothesis tests. (Two-sided test. Student–t . Significance level: α=2.5%)
TPL: The test statistic DOV/PDOV. Used to evaluate the reliability of the deflection of the vertical. Computed in north-south (X) and east-west (Y) components.
Ftest: Test statistic which is compared to Ftab. If Ftest > Ftab the H_0 hypothesis is rejected.
Ftab: Critical value used in hypothesis tests using F-distribution. (One-sided test. Significance level: α=2.5%)

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