Introduction to Vertical Reference Frames

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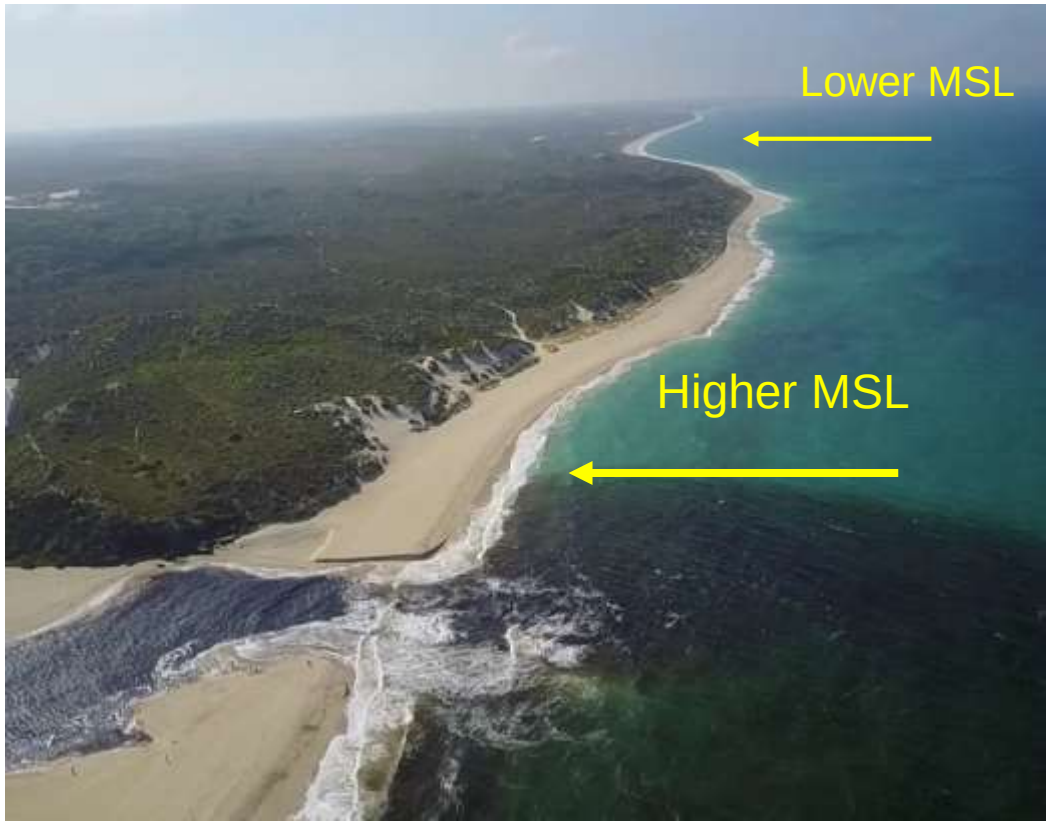
Basic Definitions

- Geodesy is the science of accurately measuring and understanding three fundamental properties of the Earth: its geometric shape, its gravity field, and its orientation in space, as well as the changes of these properties with time.
- A vertical reference frame or datum is a surface of zero elevation to which heights of various points are referred.
  - It can be defined from a base measurement point (or set of points or a horizontal surface).
  - More broadly, a vertical datum is the entire system of the zero elevation surface and methods of determining heights relative to that surface.
  - The most predominant types today are tidal datums and geodetic datums.

Datum tied to mean sea level



Example: river discharge is one modifying factor to the MSL

Mean sea surface: is the layer describing the average elevation of the sea, as is.

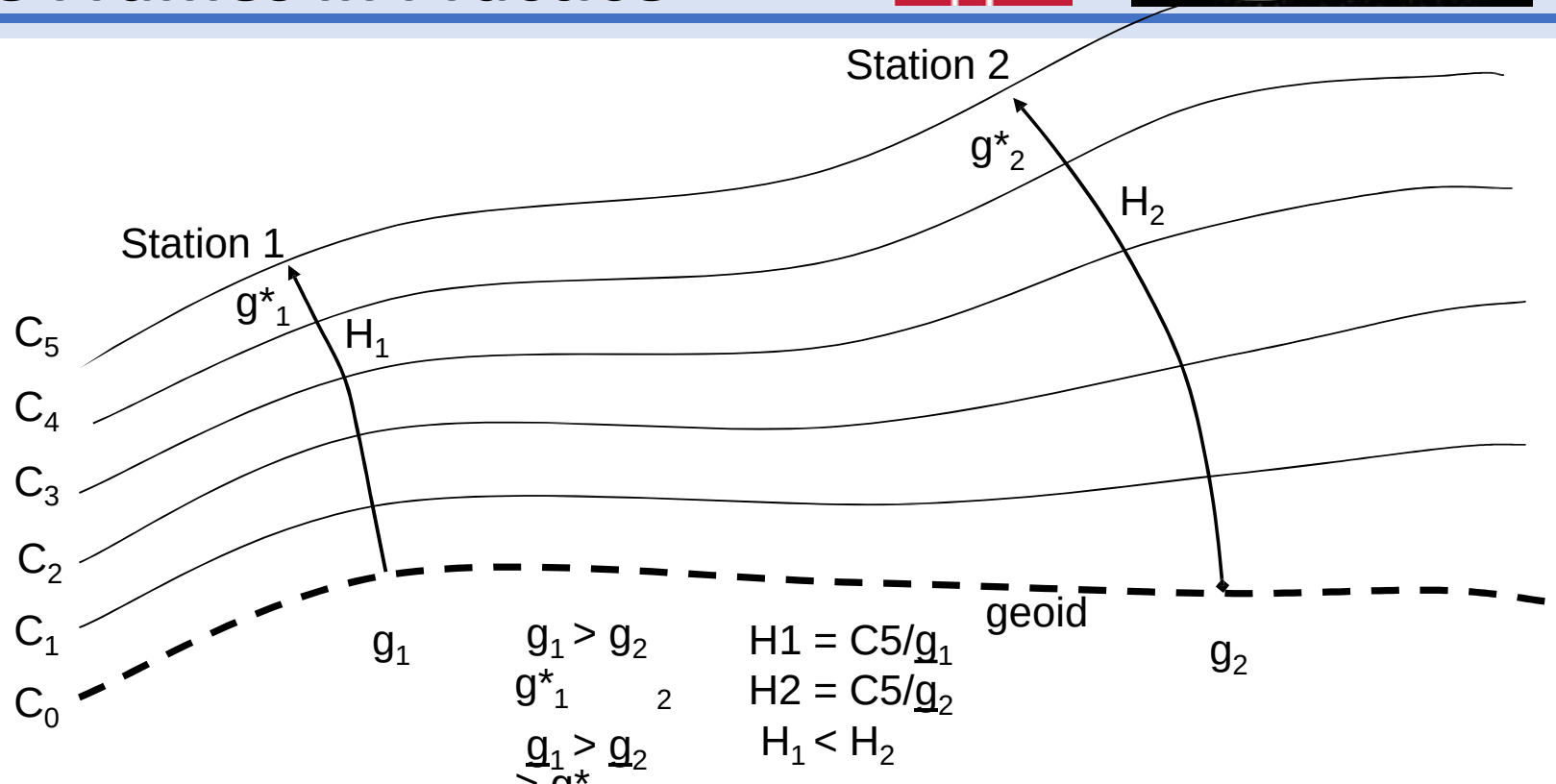
The local mean level depends on the period of observation and the environmental average local conditions like:

- atmospheric preasure,
- direction of the ocean current,
- wind direction,
- tide,
- thermohaline effect,
- river discharge,
- changing bathimetry,
- eustatic rise (climate change),
- ... and more.

What types of physical heights are there?

- Orthometric heights
  - Height (H) along the plumbline
  - Needs the integrated gravity along the plumbline
  - Directly related to surveying
- Helmert orthometric heights
  - An approximation of orthometric heights based assumptions
- Normal heights
  - Approximates actual gravity with normal (ellipsoidal) gravity
- Dynamic Heights
  - Uses a fixed gravity value
  - Effectively scales the geopotential number into meters
  - Follows the flow of water but is not consistent with surveying

Reference Frames in Practice



Note that surface location of station 1 is closer to the geoid than station 2.
A steep gradient of geops indicates higher gravity – less steep indicates lower gravity.
The geops being farther apart beneath station 2 to reflect lower local mass and gravity. Hence, H1 should be less than H2 – even though both have the same geopotential.

Distinction of concepts

Level surface:

- Determined by the gravity field as: equipotential surface.
- Only big changes in gravity can modify this surface.
- Water would not flow!

Mean sea surface:

- Can be observed by tide gauges or by satellite altimetry.
- Affected by environmental changes.
- Water would flow over a layer of MSL!

Reference Frames in Practice

A **vertical reference system** is a set of conventional definitions and parameters, adopted as a basis to determine height values which are compatible and comparable for geodetic positioning. Elements:

- Define the way Datum is reached or calculated,
- Define the type of height to be observed,
- Determine the units of measure,
- Determine the tide system,
- Determine the reference frame for positioning,
- Define the normal gravity field to be used,
- Define the way time variations are considered.

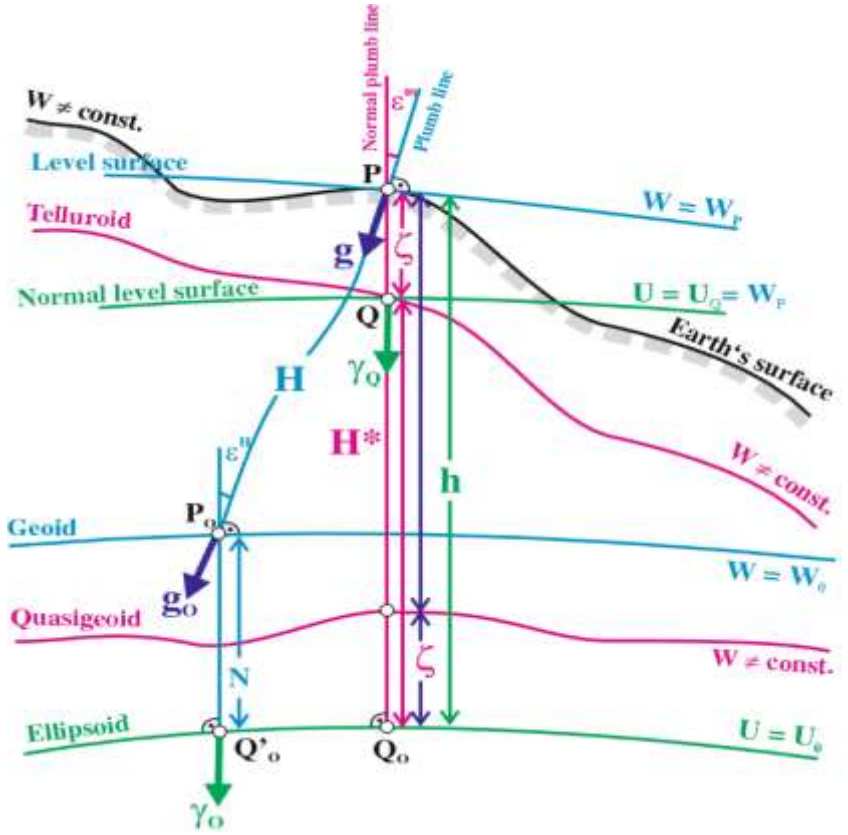


Fig. 3. From: Strategy for the realisation of the International Height Reference System (IHRS)

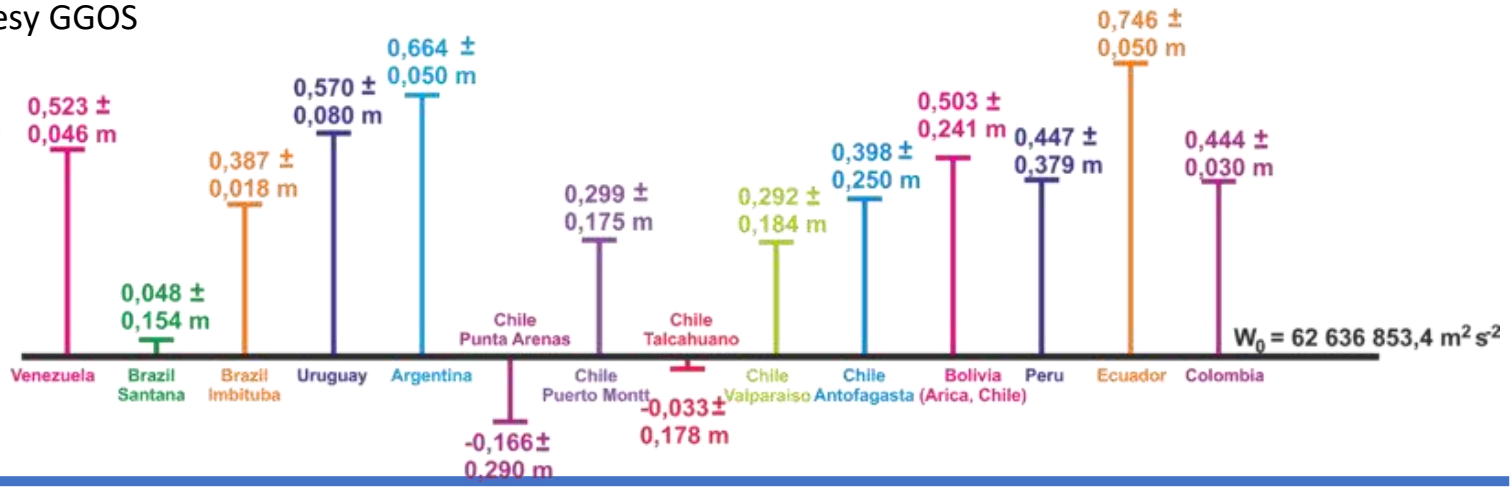
Reference Frames in Practice

A **vertical reference frame** is a realization of the system, consisting of a dataset of height values associated to a corresponding set of physical marks and some declaration of the uncertainty in such values to allow users propagate the conventional heighting measure by relative observations.

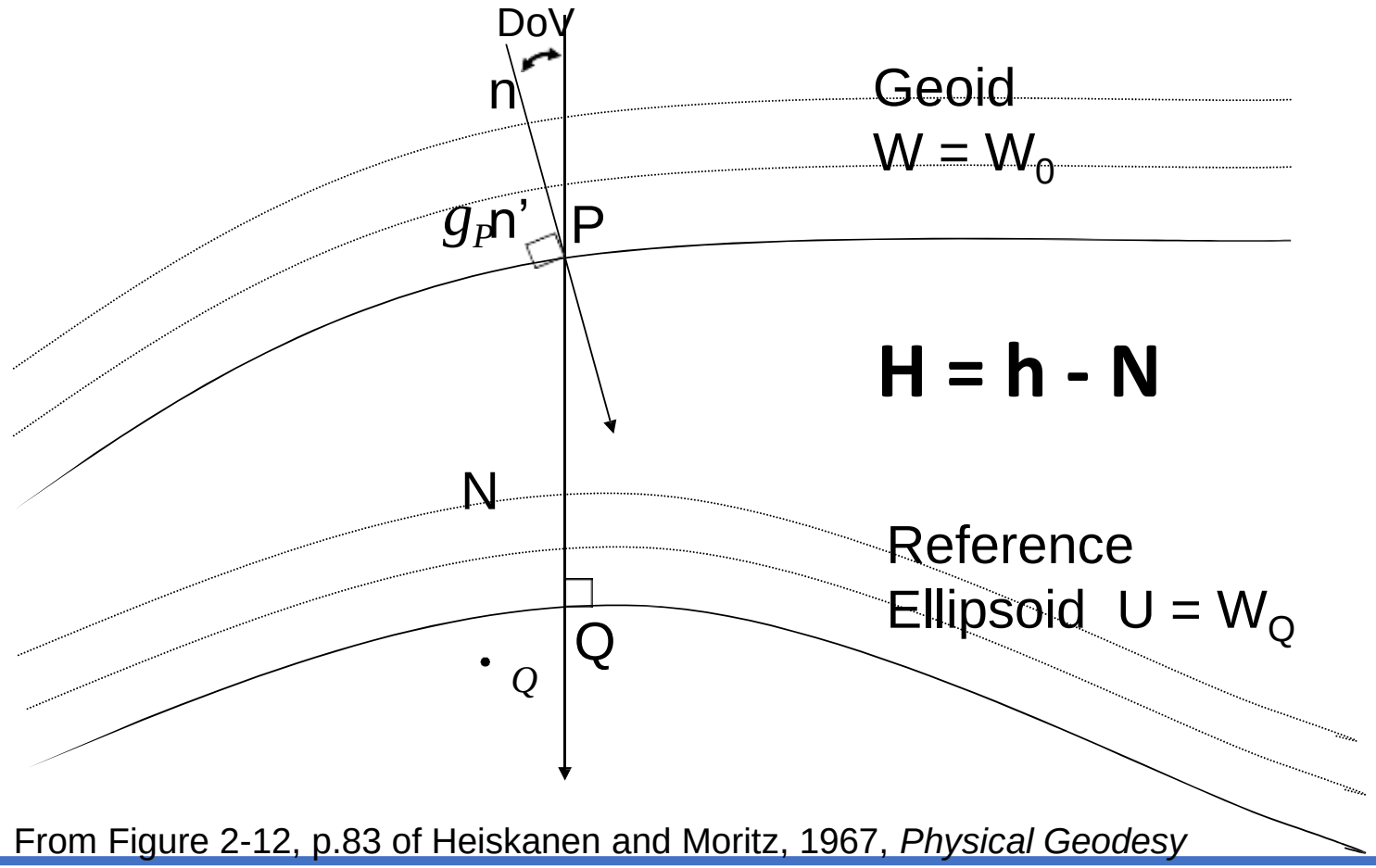
It should contain:

- Site descriptions and associated height values,
- Declaration of uncertainty,
- Description of the methodology implemented,
- An estimate of time variations in Datum or heights.

Level differences between the South American existing height systems and a global reference level. Courtesy GGOS



Gravity, Geopotential & Heights

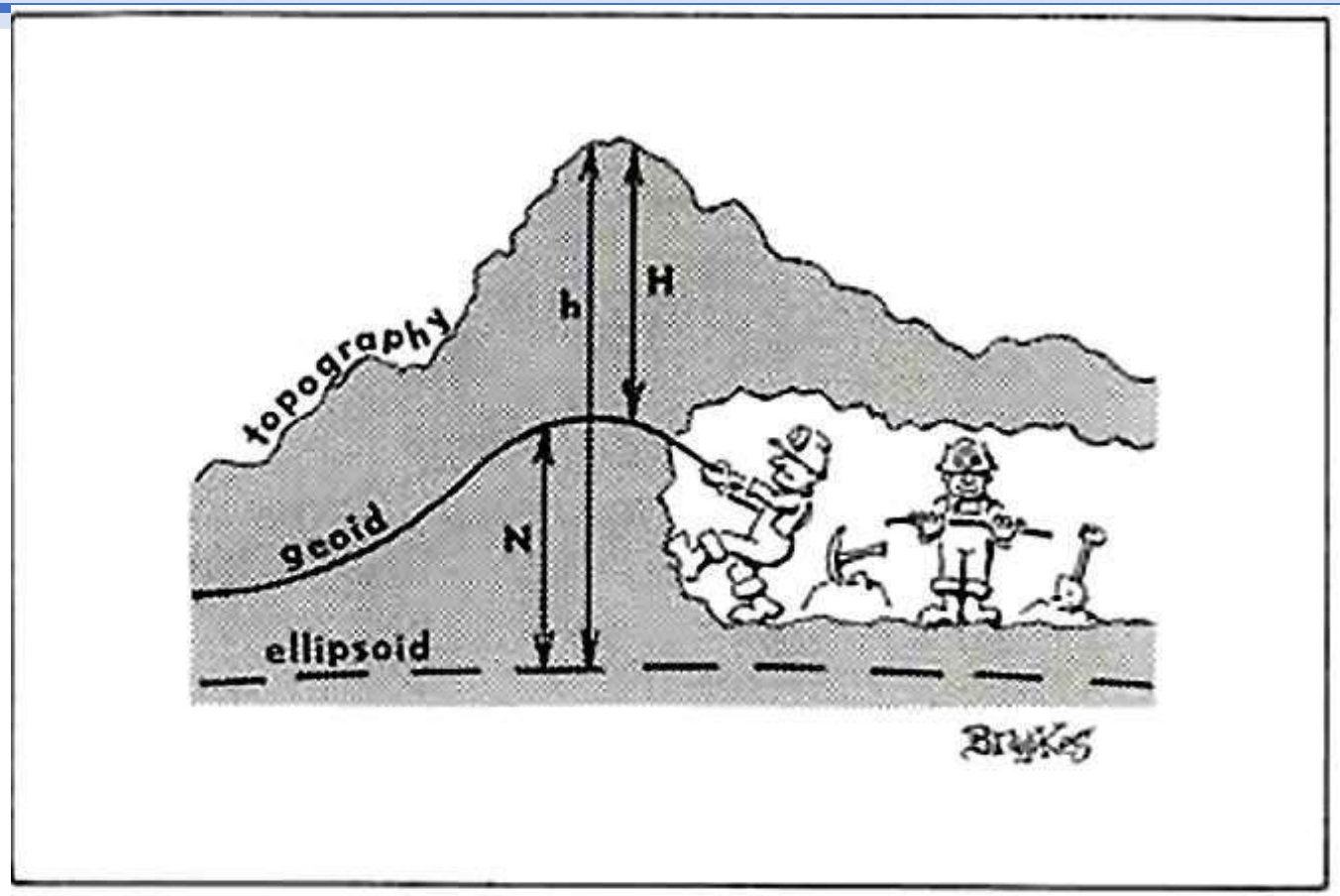


From Figure 2-12, p.83 of Heiskanen and Moritz, 1967, *Physical Geodesy*

Geoid vs. Geoid Height

- Geoid
  - The equipotential surface of the Earth's gravity field which best fits, in the least squares sense, (global) mean sea level.
  - Can't see the surface or measure it directly.
  - Can be modeled from gravity data as they are mathematically related.
  - Note that the geoid is a vertical datum surface.
- Geoid Height
  - The ellipsoidal height from an ellipsoidal datum to a geoid.
  - Hence, geoid height models are directly tied to the geoid and ellipsoid that define them (i.e., geoid height models are not interchangeable).

Reference Frames in Practice



In Search of the Geoid

Courtesy of Natural Resources Canada www.geod.nrcan.gc.ca/index_e/geodesy_e/geoid03_e.html

Datum propagation by leveling

Leveling networks are characterized by the increasing uncertainty of heights in terms of the distance reckoned from the datum point.

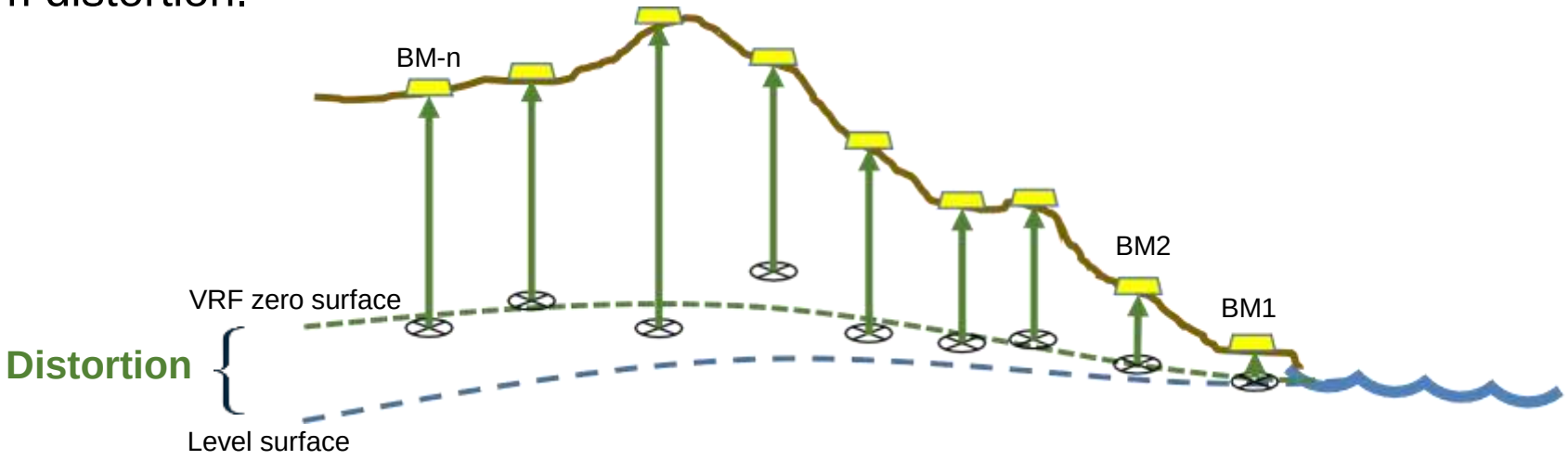
Over short distances (few tens of Km) leveling remains as the best quality technique.

Rule of thumb: Larger the distance --> larger uncertainty

Datum and distortions

The error in heights from a vertical reference frame, can be interpreted as an actual error in the datum propagation; i.e. an estimate of the datum distortion.

Datum distortion: is the systematic part of the error in the datum track.



Reference Frames in Practice

Datum and distortions

GNSS observations on pairs of benchmarks are the best source to detect **gross errors** in any of these:

- Orthometric heights,
- Geoidal heights,
- Geodetic heights.

Height differences tend to be **more reliable** than absolute height values!

Recommendation: Compare height differences!

Within error bars

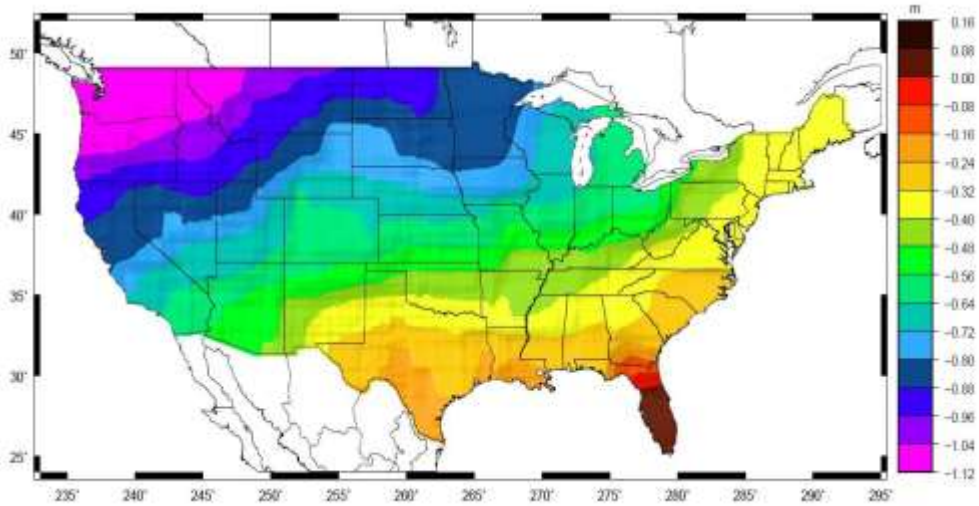
$$\Delta h = (h_2 - h_1) \approx (H_2 + N_2) - (H_1 + N_1) \approx (h_2 - N_2) - (h_1 - N_1)$$

$$\Delta H = (H_2 - H_1) \approx (h_2 - H_2) - (h_1 - H_1)$$

$$\Delta N = (N_2 - N_1) \approx$$

Datum and distortions

Depending on the extension and complexity of the study area, plus the type of gravity corrections applied to leveling, the datum distortion can grow from cm to metre level.

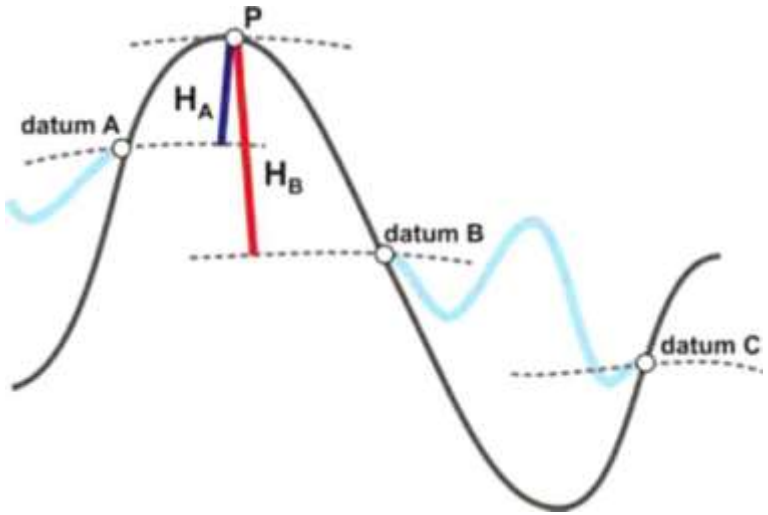


An example of well documented case of datum distortion, corresponding to the VRF named NAVD88 over the US territory.
It was measured by differencing GPS/BM geoid height versus a satellite-derived geoid of low resolution.

Datum and distortions

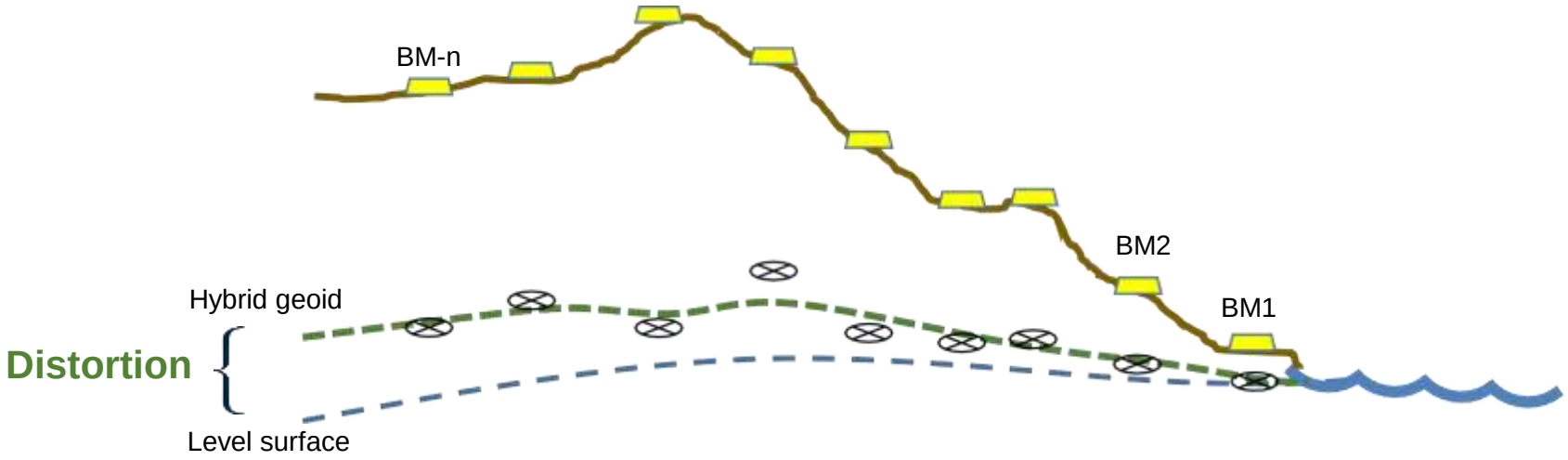
Documented disadvantages on VRF realized by leveling (geometric geoid):

- Significant datum distortions.
- May refer to different levels.
- May determine different types of height.
- Vertical variations not supported.
- Improper combination h-H-N.
- Leveling loop missclosure is treated as measurement error.



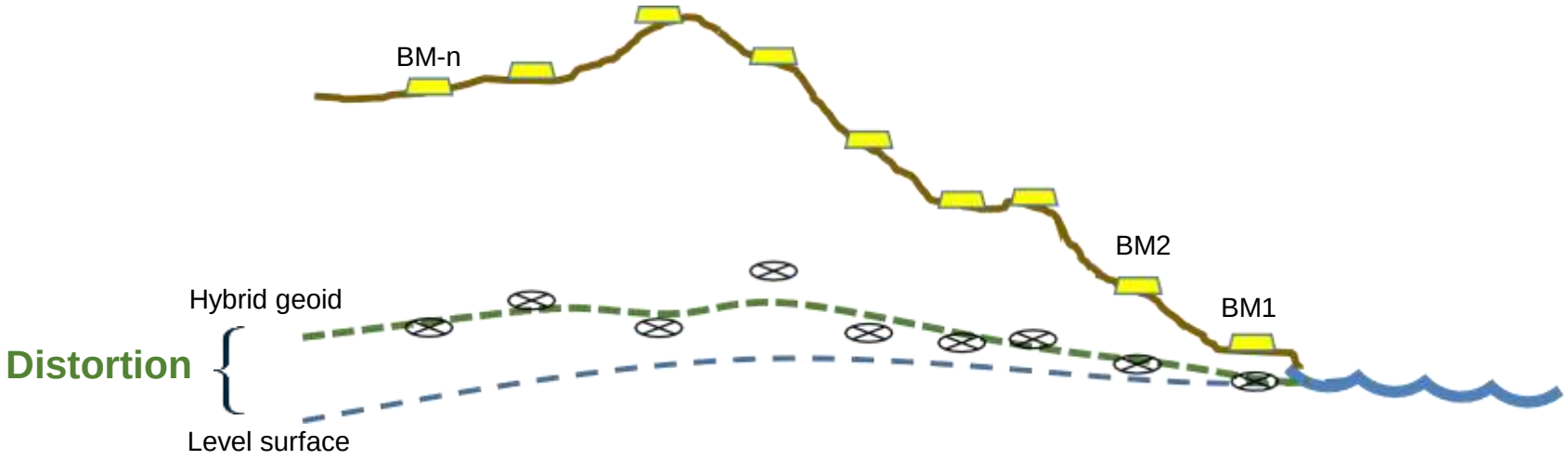
Datum and distortions

Hybrid geoid: is a geoid model produced from gravimetric information and constrained to fit the geometric geoid heights from a leveling network.



Datum and distortions

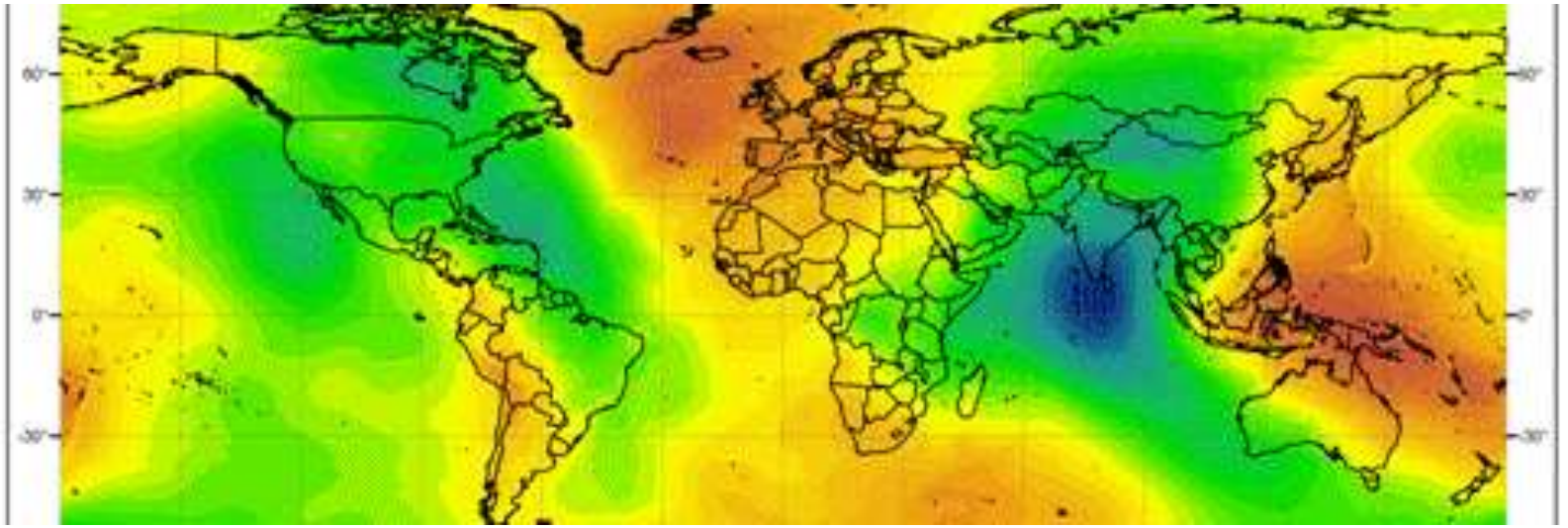
Note: hybrid geoids are characterized by a reproduction of the distortions in the leveling network.



Reference Frames in Practice

Gravimetric geoid: produced uniquely from gravimetric data and assigned to represent some predefined reference level or height.

It can span through the oceans!

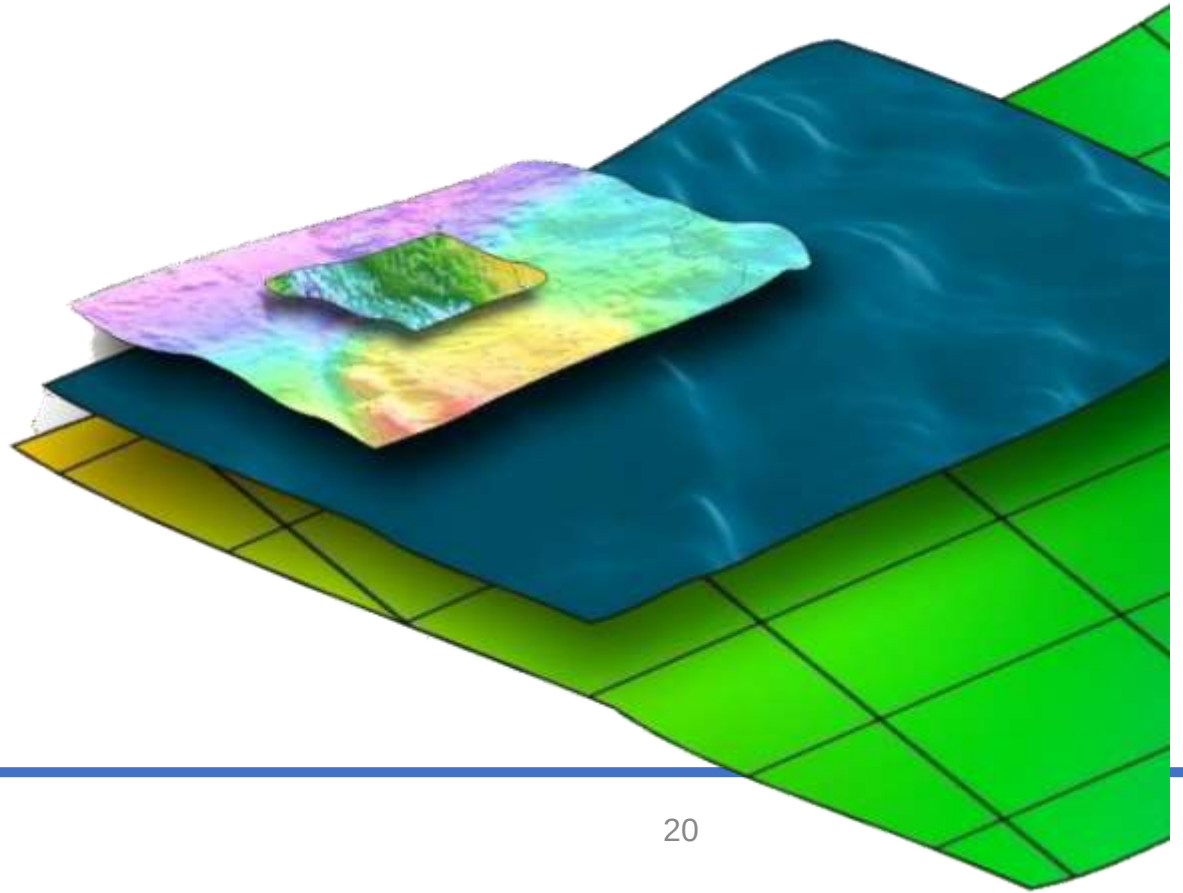


Reference Frames in Practice

A level surface

Gravimetric geoids or quasigeoids are typically built under different assumptions and parameters.

It matters to know such differences to account for the offset!



A geoid model as vertical datum

Pros:

- Permanence of the VD over time.
- No errors due to leveling observations.
- No need to re-level the whole area.
- Direct access to the VD through GNSS... faster, cheaper.

Cons:

- Terrestrial gravimetry surveys are necessary.
- Lower accuracy than leveling in relative heighting at short distances.
- Legal aspects related to other techniques or standards for heighting.

A geoid model as vertical datum

Further considerations

- It is necessary to guarantee the permanent operation of a CORS network with some specification of spatial distribution.
- The datasets available to assess the quality of precise geoid models is scarse.
- For high precission, the tide-system can be determined as "the same used in GNSS processing".
- The most popular technique to create the input gravity model is combination of satellite / terrestrial data.
- The transition from any VD to another should be accompanied by a transformation surface to allow traceability to the old standard.

Changing heights: not new.

Heights do change in time, due to land movement or datum change!

Datum changes mainly due to global sea level rise, while land can move for several reasons.

Static VRFs are unable to support height variations, and therefore, unable to guarantie high precision.

Land movement

Land subsidence produced by an earthquake (Sumatra).

Subsidence or uplift.

- Multiple reasons:
  - Ground fluids or solids extraction,
  - Tectonic activity,
  - Post-glacial rebound.

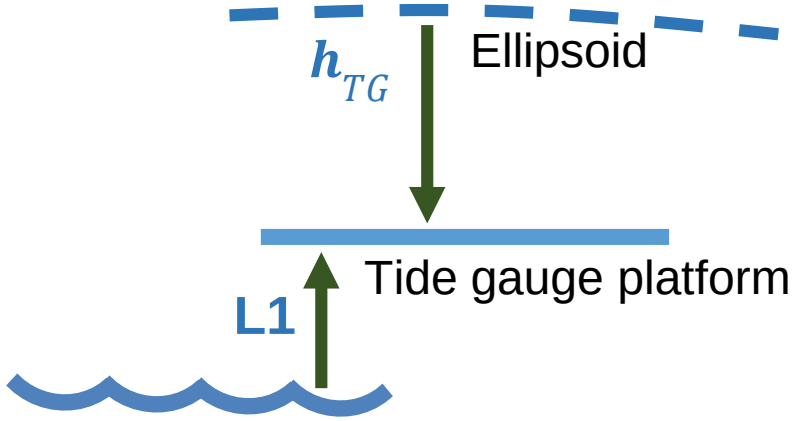
- Can be observed by:
  - GNSS repeated positioning (continuous or eventual),
  - Leveling + GNSS,
  - Satellite radar interferometry (InSAR)

Source of the vertical change

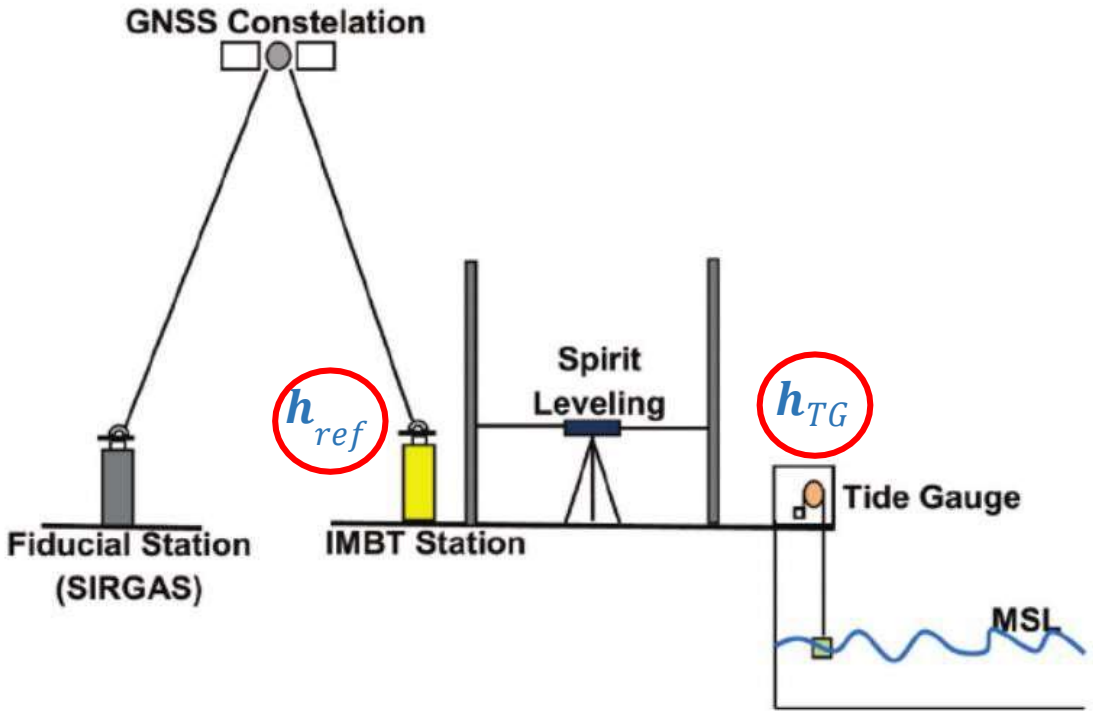
Tide gauge:
Registers the water level change with respect to the concrete platform.

GNSS:
Registers any height variation of the concrete platform with respect to the ellipsoid.

$$\frac{d}{dt}\boldsymbol{MSL} = \frac{d}{dt}\boldsymbol{h_{TG}} \quad \frac{d}{dt}\boldsymbol{L}$$



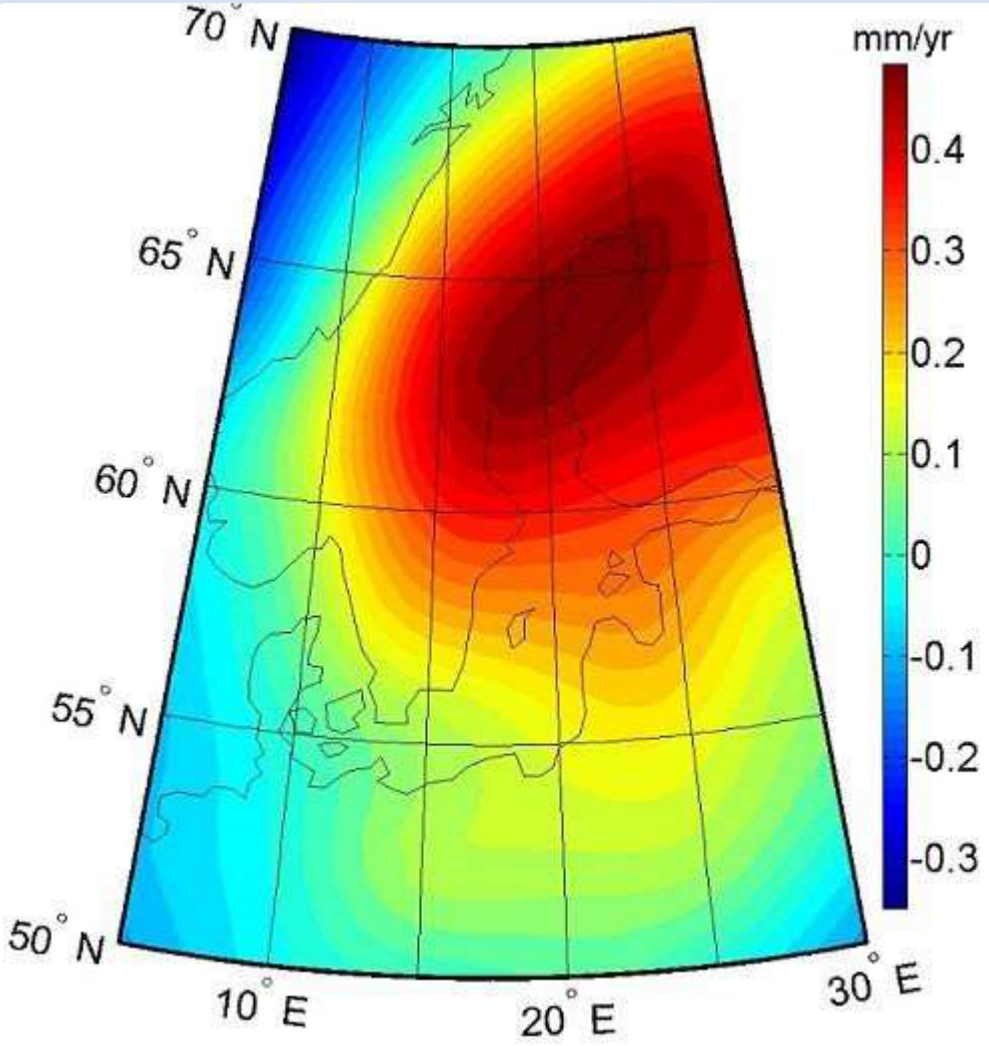
Source of the vertical change



When GNSS is not co-located on the tide gauge, a **periodic leveling** campaign helps resolving the variable dh/dt.

Reference Frames in Practice

A model dN/dt
can be provided as
component of the national
VRF to enable the support to
high precisión applications.



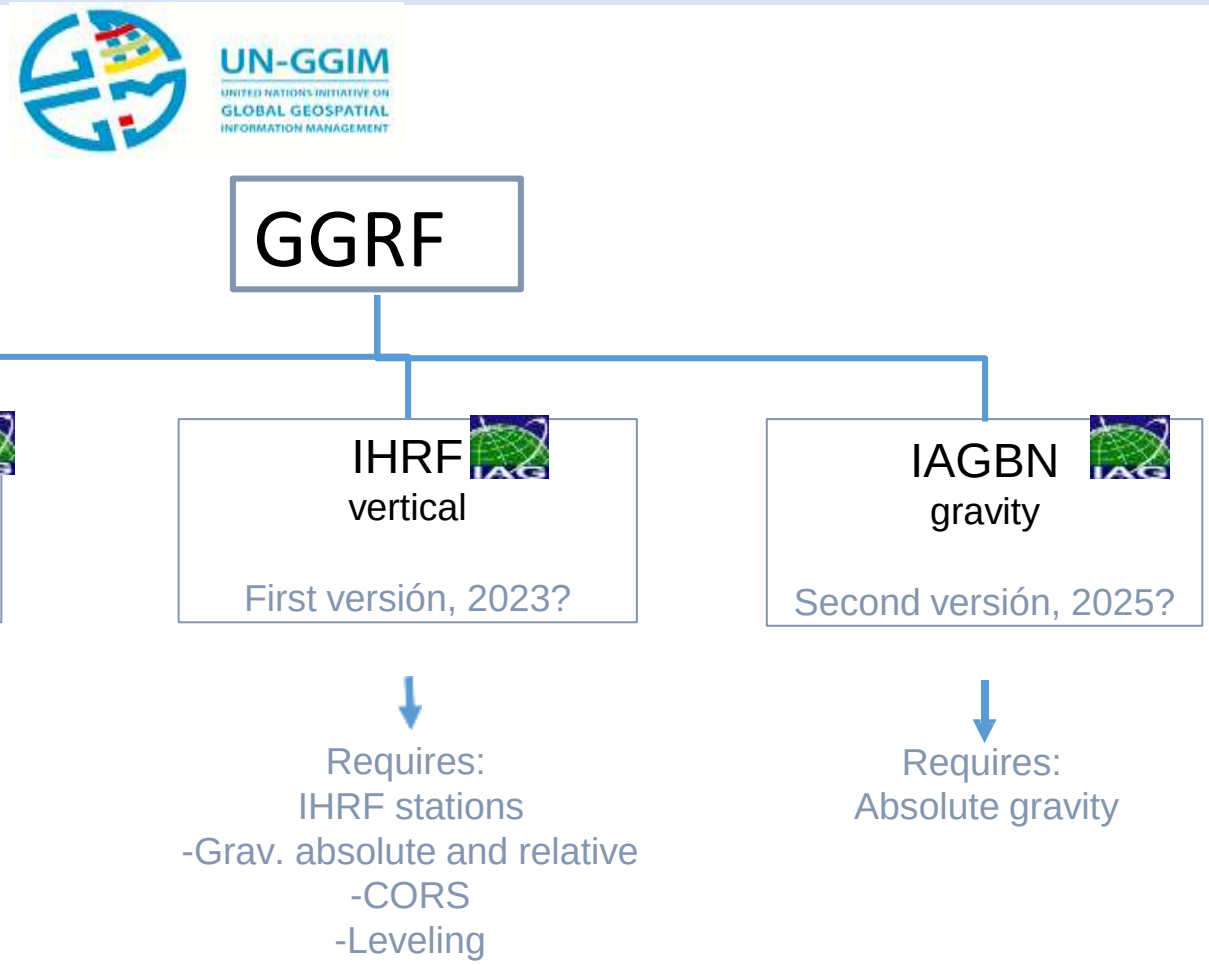
Reference Frames in Practice

From the United Nations, it has been encouraged the participation of national geodetic agencies to construct international geodetic reference frames and to adopt them as part of their geo-spatial infrastructure.

For the **vertical component** it is required to establish a series of fundamental stations to make the link between the national VRF and the international height reference system (IHRS).

Such stations are expected to support the observation of sea level change.

Reference Frames in Practice



There are more than 100 national datums, all with significant differences.

Reference Frames in Practice

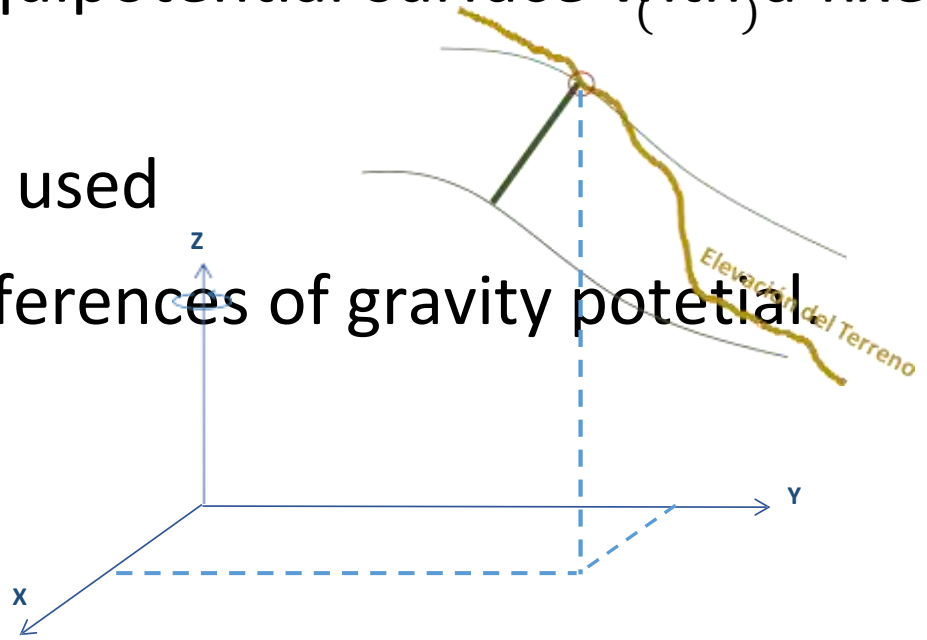
IAG Resolución, 2015:

- New reference frames are required with appropriate characteristics to research changes in the Earth.
- It is necessary to define a height system, as well as another for the gravity field.


Resolution: IHRS

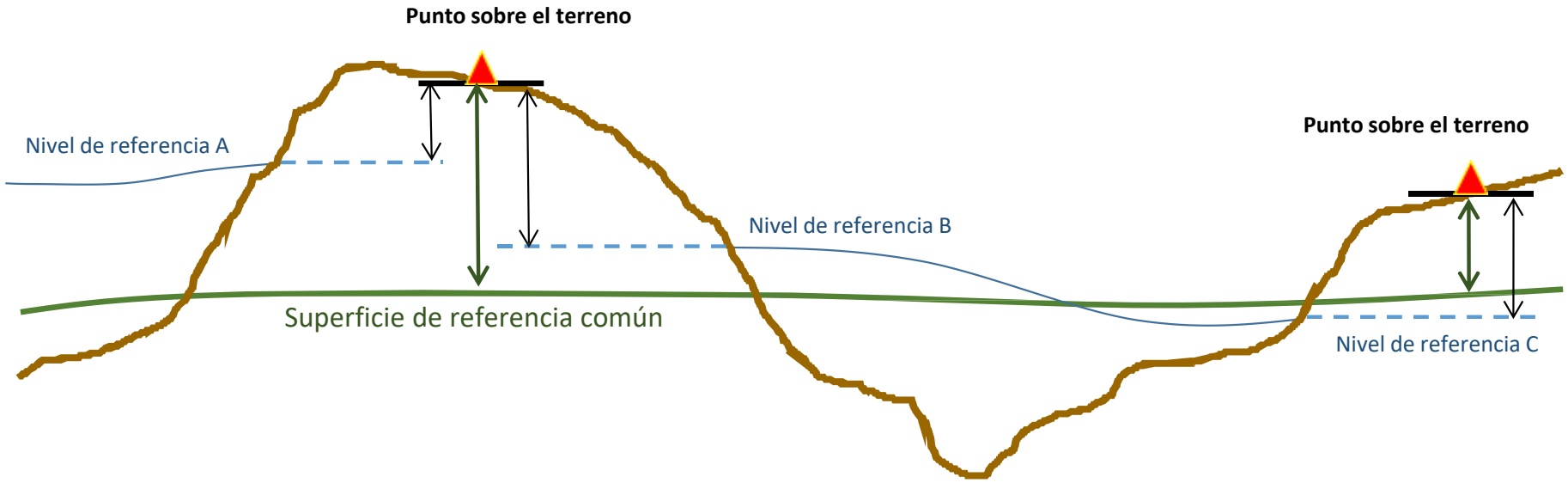
IHRS:

- The vertical reference is an equipotential surface with a fixed Wo value.
- A mean tide system should be used
- Heights are determined by differences of gravity potetial
- The standard
  - Wo = 62 636 853.4 m^2s^{-2}



IHRS

The reference level will allow the unification of height systems.



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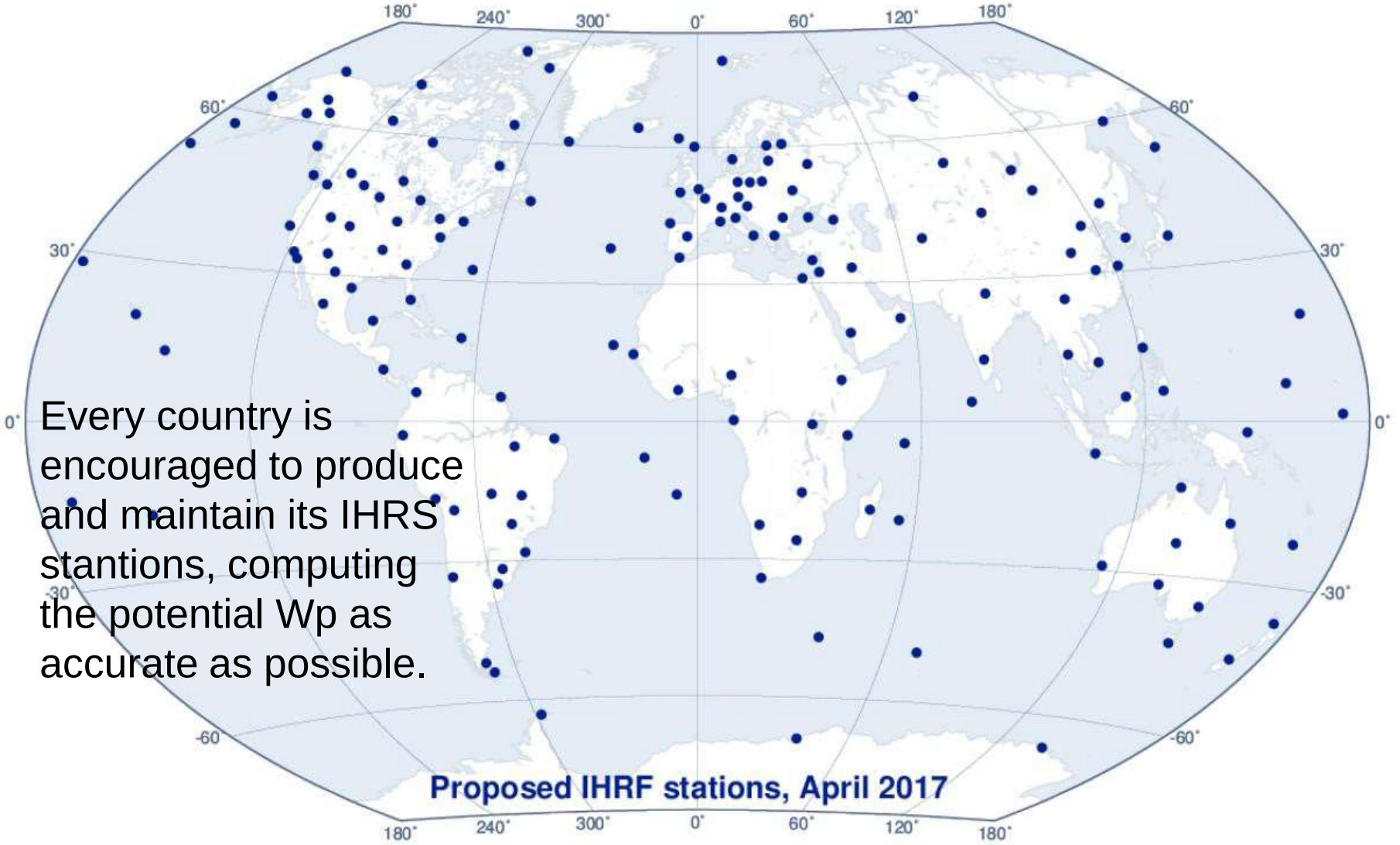


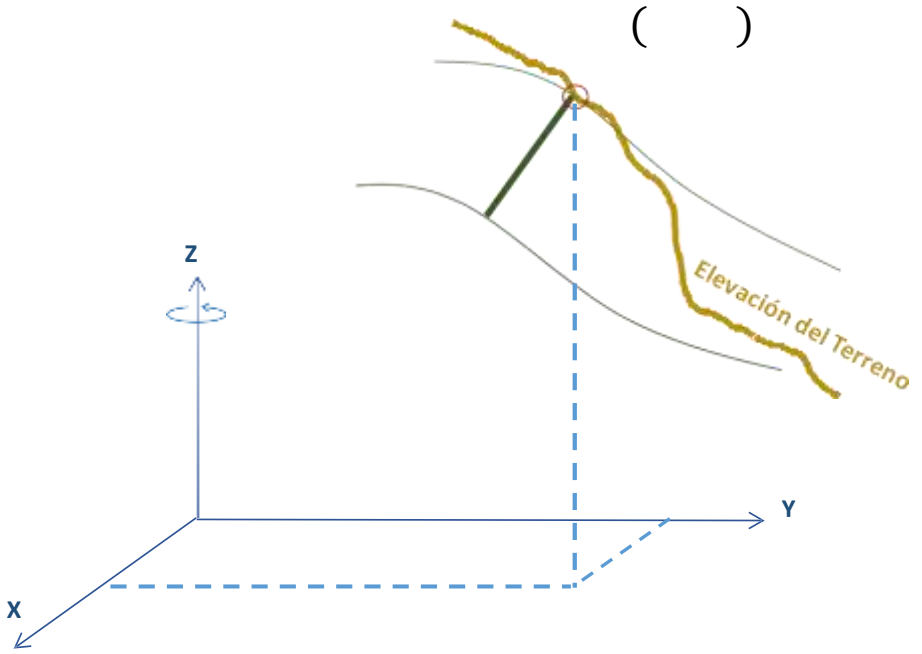
Every country is encouraged to produce and maintain its IHRS stantions, computing the potential Wp as accurate as possible.

For a point B, the geopotential number and height are:

$$C_B = W_0 - W_B = \int_0^B g\,\delta n \cong \sum_0^B g\,dn$$

$$H_B = \frac{W_0 - W_B}{\hat{g}} = \frac{C_B}{\hat{g}}$$

Every estimate of position and potential should be accompanied by the respective time variation.



In practice,

A) The Wp value can be extracted from a global geopotential model, but this is less recommended.

B) Derive Wp from existing national geoid or quasigeoid model.

$$W_P = W_{P,satellite-only} + W_{P,high-resolution}$$

B) Enhance the high resolution gravity field modeling, combining satellite and terrestrial data.

$$W_P = U_P + \gamma\,\zeta_P + \left(W_0 - U_0\right)$$

Conclusions

- Every local or national VRF contain distortions that have to be modeled and declared.
- Distortions in VRF based on gravimetric geoids/cuasigeoids tend to be random and smaller as technology advances.
- Heights can be treated as a dynamic quantity to find coherent results among surveys at different epoch.
- The world trend is to determine heights on a gravimetric reference that supports the epoch variation.

Thank you for your attention

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