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# Systematic error elimination of inertial sensors using rotating platforms

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## Abstract

Inertial measurement systems (IMSs) belong to navigation systems, which enable the determination of 3D position and trajectory independently from external signals. Functional principle of IMS is based on the integration of the output signal (acceleration and angular rate) of inertial sensors, which lead to determination of the actual position and orientation of IMS. Inertial measurements are influenced by systematic errors, which cause rapid increase of position and orientation errors in time.

The paper deals with the design of optimal model of data processing, with the aim of effective systematic error elimination in the IMS signal. For this purpose was developed the platform with two low cost IMSs, which are in controlled rotation during the system operation. Due to controlled rotation of IMSs, the orientation of the sensitive axes of inertial sensors are periodically changed and systematic errors of inertial sensors obtain a periodic character with zero mean value. This leads to the elimination of the systematic influence within each period of rotation.

The controlled rotation of the IMS's platform eliminates errors of sensors which sensitive axis is perpendicular to the rotation axis, only. The proposed algorithm enable the elimination of systematic errors of gyroscopes which sensitive axis is parallel with the IMS rotation axis. There are used two counter-rotating IMSs placed on common base, which rotate with the same angular rate. The same angular rate of IMS rotation generates the same dynamic conditions for both IMS and affects the same error in the gyroscope's signal, but with opposite sign. Results of the presented experimental measurements underlines the effective systematic error elimination in the signal of inertial sensors, based the proposed algorithm.

**Key words:** low cost IMS, systematic error, controlled rotation, common platform.

## 1 INTRODUCTION

Inertial measurement system IMS is navigation system, which consist from a group of inertial sensors and a navigation computer placed on a common platform. IMS allows continual monitoring of object's movement with high frequency rate. It is independent from external signal, what allows its using in environment, where another systems failed (indoor environment of buildings, underwater, tunnels and forests). Principle of IMS based on

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integration of inertial measurements (accelerations, angular rates) to translation and rotation movement of IMS's platform leads to rapid accumulation of systematic errors of inertial measurements in actual position and orientation (Groves, 2008) (Fig. 1).

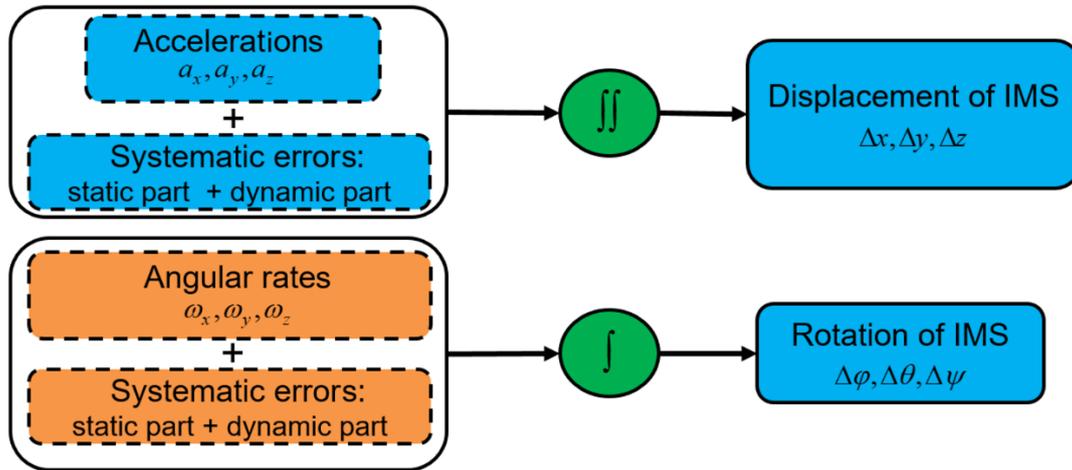


Fig. 1 Accumulation of systematic errors of IMS's signal in integration process

Systematic error of inertial measurement consist from static and dynamic part. Static part of systematic error (SPSE) can be eliminated by using error's model of inertial sensors, which was defined by calibration of used sensor. The dynamic part of systematic error (DPSE) can be not eliminated by calibration, because its change during measurement in dependence on dynamic of IMS's movement. The presence of a DPSE in the IMS's signals leads to an increase position and orientation errors with the increasing time of measurement (Fig. 2). This effect is multiplied by increasing the frequency rate.

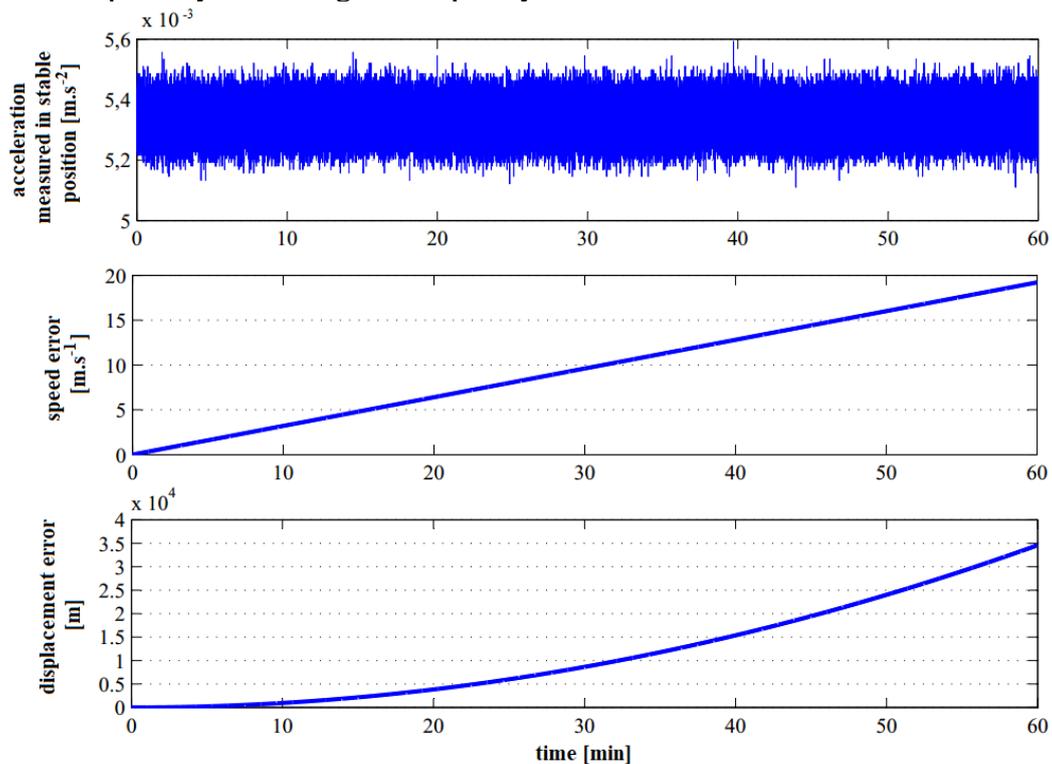


Fig. 2 Increase velocity and displacement error generated by double integration of the measured acceleration (time series represent one hour of measurement logging with a frequency rate of 100 Hz)

Zero velocity update (ZUPT) is often used to eliminate systematic errors of accelerometer. The principle of this method is based on condition of the zero velocity at a time, when IMS is stable. Based on this condition, the error's component of the calculated velocity is modelled as a linear function defined (calculated by integration of an acceleration) at the moment of starting and stopping the IMS.

ZUPT allows to eliminate only a linear part of error's components of the velocity (Fig. 3). Nonlinear part of error's components of velocity generated by change of dynamic of IMS's movement is accumulated to distance errors, which dependence on length of time interval between the stops of the IMS.

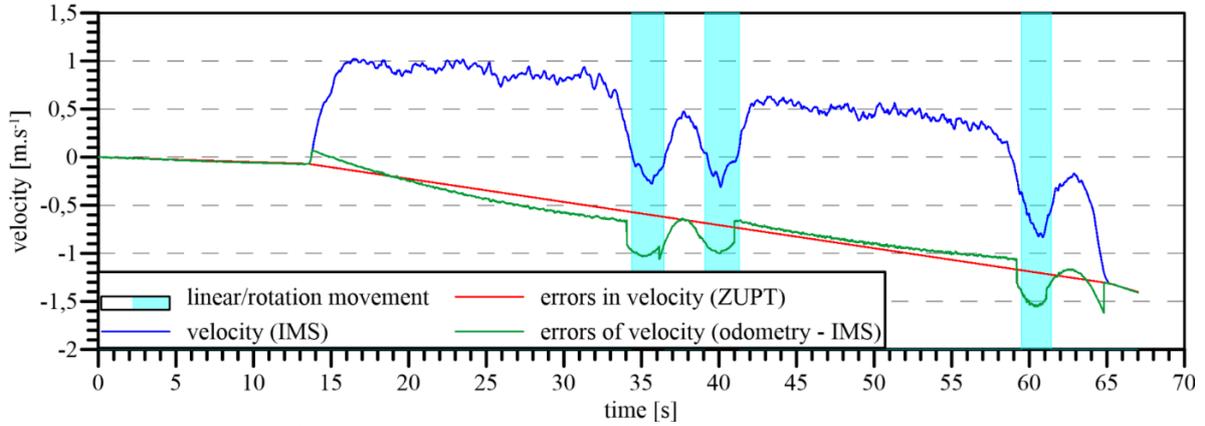


Fig. 3 Change of the error's component of calculated velocity affected by change of dynamic state of the IMS

The increase position error resulting from insufficient elimination of DPSE of accelerometers with respect to the recursive algorithm for position determination. The solution to the problem is a combination of IMS with an additional sensor (for example encoder), which allows used measurements at constant time intervals to calculate the error's component (as opposed to ZUPT in which it uses measurements/conditions only at the time, when IMS stops).

## 2 ELIMINATION OF SYSTEMATIC ERRORS OF INERTIAL SENSORS USING CONTROLLED ROTATION OF IMS

A new approach to eliminating the systematic errors of inertial sensors is based on using the rotational motion of the IMS's platform (Ben, 2011), (Collin, 2015), (Sun, 2012). Rotating inertial measurement system (RIMS) generate periodical change of orientation of sensitive axis of inertial sensors, what leads to periodical change of DPSE with zero mean value (after eliminate centrifugal acceleration generated by rotation). Periodic change of value (and sign) of DPSE leads to its elimination within each rotation period.

$$R_{IMS}^R \cdot \varepsilon_{\omega_{IMS}} = \begin{pmatrix} \cos(\omega_r \cdot t) & \sin(\omega_r \cdot t) & 0 \\ -\sin(\omega_r \cdot t) & \cos(\omega_r \cdot t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{\omega_x} \\ \varepsilon_{\omega_y} \\ \varepsilon_{\omega_z} \end{pmatrix} = \begin{pmatrix} \varepsilon_{\omega_x} \cdot \cos(\omega_r \cdot t) + \varepsilon_{\omega_y} \cdot \sin(\omega_r \cdot t) \\ -\varepsilon_{\omega_x} \cdot \sin(\omega_r \cdot t) + \varepsilon_{\omega_y} \cdot \cos(\omega_r \cdot t) \\ \varepsilon_{\omega_z} \end{pmatrix} \quad (1)$$

$$R_{IMS}^R \cdot \varepsilon_{a_{IMS}} = \begin{pmatrix} \cos(\omega_r \cdot t) & \sin(\omega_r \cdot t) & 0 \\ -\sin(\omega_r \cdot t) & \cos(\omega_r \cdot t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{a_x} \\ \varepsilon_{a_y} \\ \varepsilon_{a_z} \end{pmatrix} = \begin{pmatrix} \varepsilon_{a_x} \cdot \cos(\omega_r \cdot t) + \varepsilon_{a_y} \cdot \sin(\omega_r \cdot t) \\ -\varepsilon_{a_x} \cdot \sin(\omega_r \cdot t) + \varepsilon_{a_y} \cdot \cos(\omega_r \cdot t) \\ \varepsilon_{a_z} \end{pmatrix} \quad (2)$$

where:

- $\omega_r$  - angular rate of the controlled rotation of IMS,
- $\varepsilon_{a_{IMS}}, \varepsilon_{\omega_{IMS}}$  - systematic error of acceleration/angular rate.

Periodical change of DPSE during rotation of IMS is defined by a rotation matrix  $R_{IMS}^R$ , which represent the rotation of coordinate system of IMS  $CS^{IMS}$  with the respect to reference coordinate system  $CS^R$ :

The controlled rotation of IMS allows to eliminate the DPSE of sensors which sensitive axis is perpendicular to the rotation axis of the IMS. When IMS rotates around their Z axis (or axis perpendicular to plane of the IMS's movement) could be not eliminated DPSE of sensor with sensitive axis parallel with the IMS rotation axis, what leads to increasing errors in the system orientation.

### 3 DESIGN OF THE MODEL OF DATA PROCESSING

The proposed model of data processing (Fig. 4) combines several procedures for eliminating systematic errors, namely:

- Application of Error Models of used sensors (SPSE elimination).
- Controlled rotation of IMS (elimination of DPSE from inertial measurement in the direction of the axis perpendicular to the axis of rotation).
- Using a pair of counter-rotating RIMS used to elimination DPSE of gyroscope's measurement in the direction of the axis parallel to rotation axis.

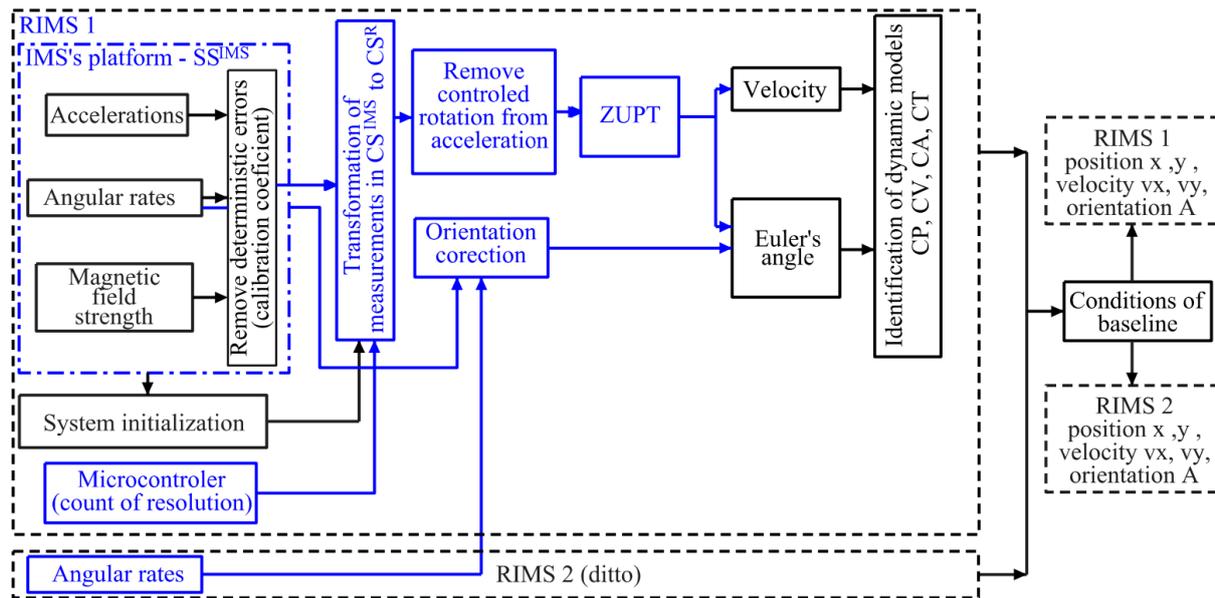


Fig. 4 Scheme of proposed model

The data processing of inertial measurements made by RIMS is different from processing of measurements from IMS with stable platform, which is described in article (Kopáček, 2016). Resulting the controlled rotation, the orientation of the  $CS^{IMS}$  continually changes with respect to  $CS^R$ . Therefore, the model for data processing was completed by transformation of measurements realized in  $CS^{IMS}$  into the  $CS^R$ , whereby the rotational angle was calculated from the angular velocity of the IMS rotation.

In the next step, it is necessary to eliminate centrifugal accelerations (generated by controlled rotation) from the measured accelerations, using Butterworth bandpass filter. The selection of this filter was conditioned by its maximally flat frequency response in the passband. The frequency range of the filter is defined based on the angular rate of the IMS platform rotation (Fig. 5):

$$f_{rot} = \frac{\omega_{rot}}{2\pi} \tag{3}$$

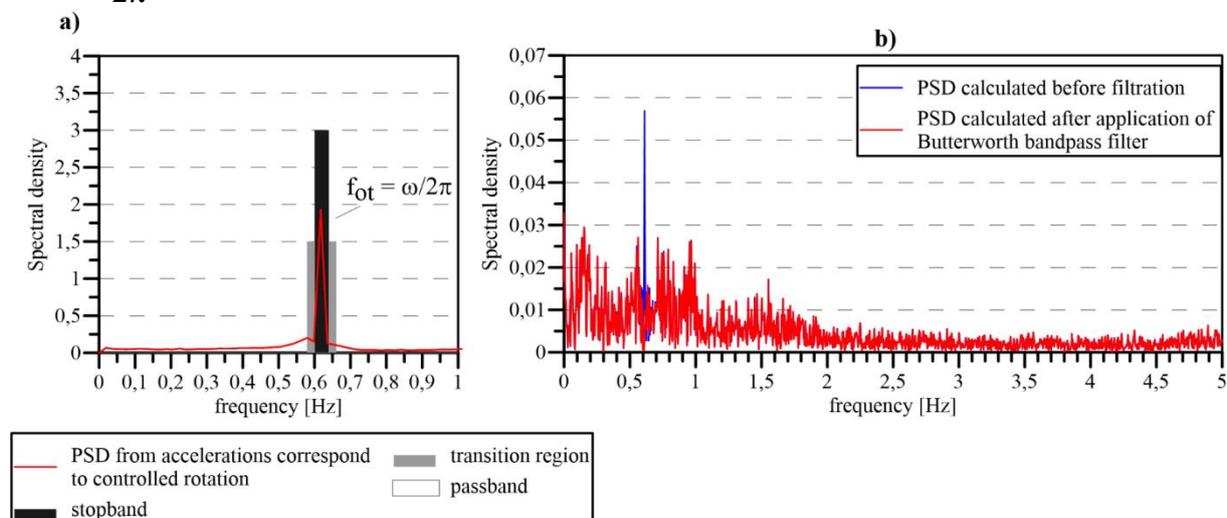


Fig. 5 Elimination of centrifugal accelerations from measured signal with using Butterworth bandpass filter (left – frequency response of designed filter, right – power spectral density PSD of signal before and after using a filter)

As was mentioned before, the RIMS does not eliminate the systematic errors of sensor which sensitive axis is parallel with the rotation axis, what leads to accumulation of errors by these sensors. During rotation of the IMS around the Z axis (or axis perpendicular to plane of RIMS movement), are affected by this the gyroscopes, which accumulate errors in the system during they movement. To eliminate these kind of errors and the DPSE of the gyroscopes in the direction of the rotation axis is used a pair of counter-rotating IMS platforms. Using a pair of counter-rotating RIMS generate the same dynamic state of both RIMS (due to the same angular rate of rotation), which influence the course of DPSE.

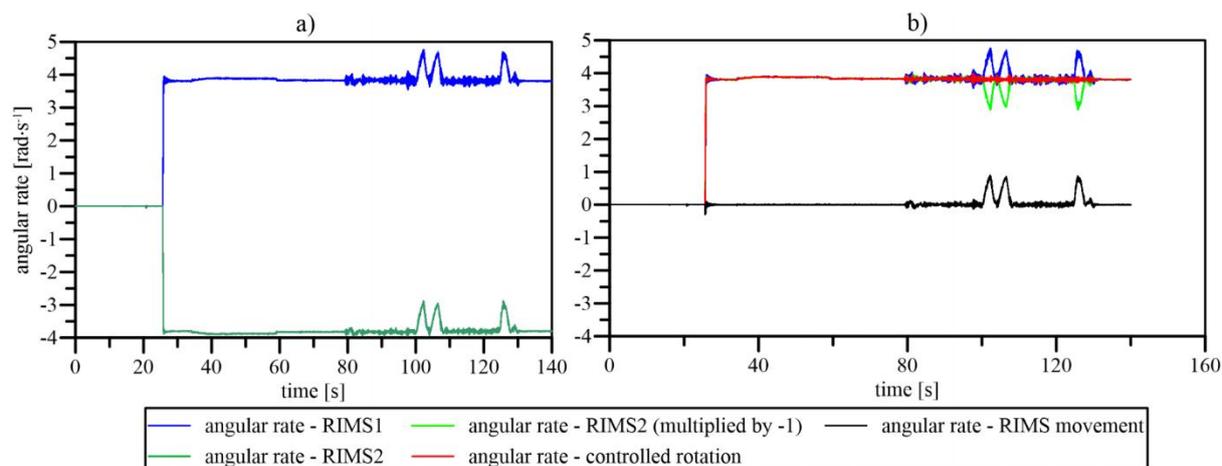


Fig. 6 Elimination of DPSE of gyroscope in Z axis generated by controlled rotation of RIMS (a – measured angular rate, b – separation of angular rate on component corresponding to controlled rotation and movement of measured system)

The arithmetic mean of the angular velocities of both RIMS  $\omega_Z^1, \omega_Z^2$  (formula 4) expresses an angular velocity corresponding to the controlled rotation of the RIMS's platform with the DPSE of angular rate (Fig. 6b - the red line). In calculating the arithmetic mean, the angular velocity of the second RIMS  $\omega_Z^2$  is explicitly assigned the opposite sign.

The angular rate corresponding to the movement of the measuring system (Fig. 6b - black line) is calculated by subtracting the angular velocity of controlled rotation from the angular rate measured by the RIMS:

$$\omega_{\text{rot}} + \varepsilon_{\omega} = (\omega_Z^1 + (-\omega_Z^2))/2 \quad (4)$$

$$\omega_{\text{poh}} = \omega_Z^1 - \omega_{\text{rot}} + \varepsilon_{\omega}. \quad (5)$$

#### 4 EXPERIMENTAL MEASUREMENT AND VERIFICATION OF THE PROPOSED MODEL

In order to testing the proposed model, a measuring system was designed. Measuring system consists from trolley with the baseline. At both ends of the baseline was placed RIMS. Controlled counter-rotating movement of RIMSs with the same angular rate was realized by a pair of stepper motors, which are controlled by a common microcontroller. The measuring system was supplemented by an optical rotary encoder, which served to analyse DPSE of accelerometers.

During the experimental measurements, the measurement system was moved along predefined trajectory with rectangular shape. The trajectory consists from straight lines, where transition between the straight lines of the trajectory was realized by arc. The 9 control points (CP) of trajectory were located on intersection of the straight line with arc. These points were used to analyse the efficiency of the proposed model for elimination of DPSE of inertial sensors.

The resulting error in position consists from two parts – error in the determination of the trajectory length (generated by DPSE of accelerometers) and the error in orientation of RIMS's (generated by the DPSE of gyroscopes). The elimination of accelerometers DPSE was analysed based on the differences between the reference and measured distance between trajectory CPs. In the case of gyroscopes, the model efficiency was analysed based on differences in the orientation of the straight part of the trajectory. Positioning accuracy was analysed based on a relative positional difference  $\Delta\delta$ , which is defined as the ratio between difference in position of the selected CP  $\Delta p$  and the corresponding length of travelled trajectory  $d_{TT}$ :

$$\Delta\delta = \frac{\Delta p}{d_{TT}}. \quad (6)$$

According the experimental results (Fig. 7) could be concluded, that the ZUPT eliminates only the linear part of the accelerometers DPSE, what leads to increasing error in measured distance with the increased length of the time interval between RIMS stops (average difference achieved during the experiment was up to 0.7 m).

When using the RIMS to eliminate DPSE from measured accelerations resulting to average relative position difference 0.009, which correspond to position difference 9 mm / 1 m of travelled trajectory. This is comparable with the combination of IMS and optical encoder, where the average relative position difference is 0.005. Therefore, the use of RIMS can be understood as an alternative solution in case, when the optical encoder cannot be used. Increasing the angular rate of RIMS leads to more effective elimination of DPSE of inertial sensors, because period of rotation movement is shortened. During short periods, there can be assumed that the DPSE will not change significantly.

Larger differences in distances calculated from model and the RIMS are affected by the slowly adaptation of the frequency filter, which is applied to remove the centrifugal acceleration corresponding to rotation movement of RIMS. Changes in the angular rate of RIMS rotation negatively affect to accuracy of the calculated position.

The use of a pair of counter-rotating RIMS allows efficiently eliminate DPSE of gyroscopes in the direction of the axis of rotation, which significantly increased the accuracy of orientation of the RIMS movement (orientation drift is  $0.0015 \text{ }^\circ\cdot\text{s}^{-1}$ ) compared to the use of one RIMS (orientation drift is  $0.0065 \text{ }^\circ\cdot\text{s}^{-1}$ ).

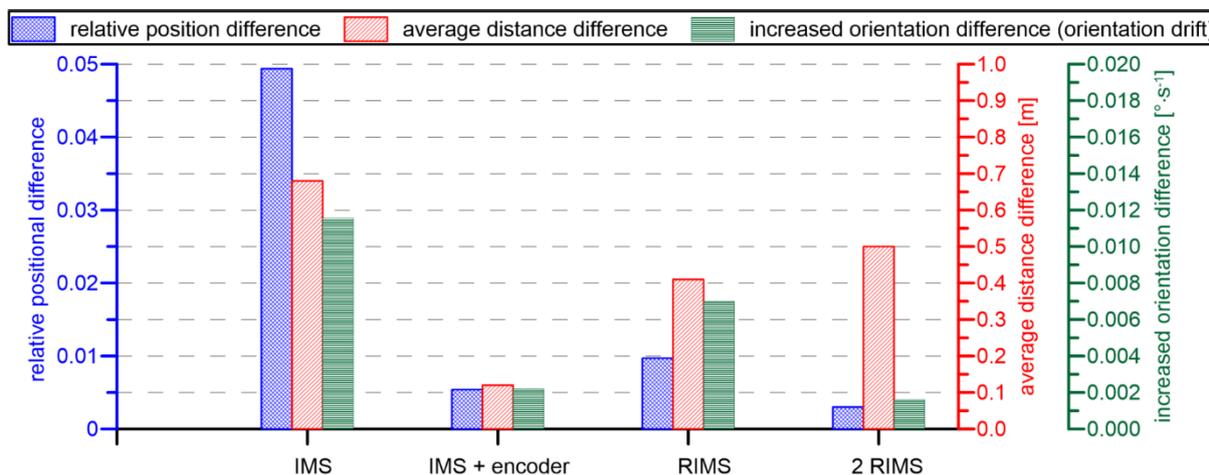


Fig. 7 Comparison the effectiveness of different approaches for systematic error elimination in the IMS's signal

## 5 CONCLUSION

Thanks to the advantages of IMS (high frequency rate, signal independence from external sources) compared to other navigation systems (GNSS, ultrasound), it has a significant role in monitoring the position of the object and its dynamic properties. The application of IMSs in geodesy are limited by the increasing position and orientation error, due to accumulation of systematic errors of inertial sensors in the integration process.

This article brings a new approach to eliminating systematic errors of inertial sensors, based on rotation of IMS's platform. Controlled rotation of IMS generates a periodic change of IMS's signal what leads to periodic course of DSPE. During the short period of IMS rotation, the DSPE does not significantly change, what resulting to its periodic course and enable their elimination. With increasing angular rate of RIMS rotation could be the DPSE eliminate more efficiently.

Using only one RIMS, it is not possible to eliminate a systematic error of inertial sensors, which sensitive axis is parallel to the axis of rotation of the RIMS. In order to solve this problem, an algorithm using a pair of counter-rotating RIMS's was designed. The principle of the proposed algorithm is based on the assumption that the same dynamic conditions of both RIMS (generated by the same angular rate of both RIMS) leads to same course of DPSE of gyroscopes.

The results of experimental measurements show a significant benefits of the proposed model for inertial sensors DPSE elimination. RIMS is a suitable alternative to combination of IMS with optical encoder in case, when optical encoder is not possible to use.

Using the pair of counter-rotating IMSs, the proposed model of data processing effectively eliminates the DPSE of gyroscopes, what leads to increased precision in determined orientation of the system movement as well as increasing precision in the determined position.

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