Stability Determination of the Surface Area of the Prague Castle by the Periodically Measured Levelling Network and Robust Analysis

Štroner, M., Urban, R. and Kubín, T.

Czech Technical University in Prague, Faculty of Civil Engineering, Thákurova 7, 166 29 Prague, Czech Republic, Web site: www.fsv.cvut.cz
E-mail: martin.stroner@fsv.cvut.cz, rudolf.urban@fsv.cvut.cz, tomas.kubin@fsv.cvut.cz.

Abstract

In the area of Prague Castle there is already about 10 years of periodical measurement of the height changes in the buildings and structures being performed. The measurement method is precise levelling predominantly. Until now, these measurements have been evaluated only locally with respect to each building and its stability without an overall view of the situation of possible movements of individual parts of the surface Prague Castle. Whereas there are height shift of some points between epochs undoubtedly, a new and complete adjustment of each measured epoch and mutual assessment of changes between epochs using robust analysis was conducted. This comparison shows the relative movement of certain parts against another. The results are consistent with current knowledge of the geology in the area of the Prague Castle.

Key words: Robust Analysis, Deformation Analysis, Precise Levelling, Prague Castle

1 INTRODUCTION

Prague Castle is one of the most important historical, political and tourist areas of the Czech Republic, since 1918 also the seat of the President of the Czech Republic.

According to (Prague Castle, 2014), the Prague Castle complex was created by sequential additions and renovations of the settlement founded in the 9th century. With its dimensions of 570 m length and 128 m width it is one of the largest castle in the world. It is considered to be not only symbol of the city, but also the Czech statehood. Historic buildings located in the area are however affected by the aging process and the effects of changes in the surroundings. In order to predict further developments in this area, the long term periodic measurements for determining the stability of historic buildings in the area of the Prague Castle are carried out.

Geology in the area plays major role, according to (Zálesky, 2001) it was originally not complicated, but anthropogenic activities related to structural modifications of Hradčany hill during the last centuries made it considerably more complicated. The bedrock of the area has been reworked and expanded by the fills of different origin. The first measuring was conducted by the Department of Special Geodesy in 1999, since then it is still ongoing and have been supported by several grants. The findings and conclusions of the measurements were summarized in (Procházka, 2011). These measurements were initially concentrated on the fault monitoring of individual buildings, and later connected via a network of reference points for both height measurements (precise levelling) and the position measurement. But because of this non-systematic evolution of the measurement there are differences between epochs in configuration of the network and of monitored points, according to actual demand.

So far these measurements were evaluated only locally with respect to each building and it's stability without an overall view of the situation of possible movements of individual parts of the surface of the Prague Castle area. A comprehensive evaluation of the height shifts is further discussed in this article.

2 GEODETIC MEASUREMENTS AT PRAGUE CASTLE

Geodetic measurements were in the area of Prague Castle carried out in various range since its construction, but the periodic monitoring of selected historic buildings and slope stability is a matter of the last 10 years. There are changes monitored in tilt and relative height in individual buildings and areas.

For analysis only height measurement was chosen. The reason for this decision is a very high and long-term achieved precision of 0.7 mm / km, also the high reliability and resistance of the method to systematic errors. Height measurements are almost entirely a matter of method levelling from the centre with the addition of precise trigonometric method used to bridge the Jelení příkop (Deer Moat). Scheme of the performed measurements is on Fig. 1. Measurements were conducted with use of the Zeiss Koni 007 instrument, and in the last two epochs the digital levelling instrument Trimble Dini 12T was used.

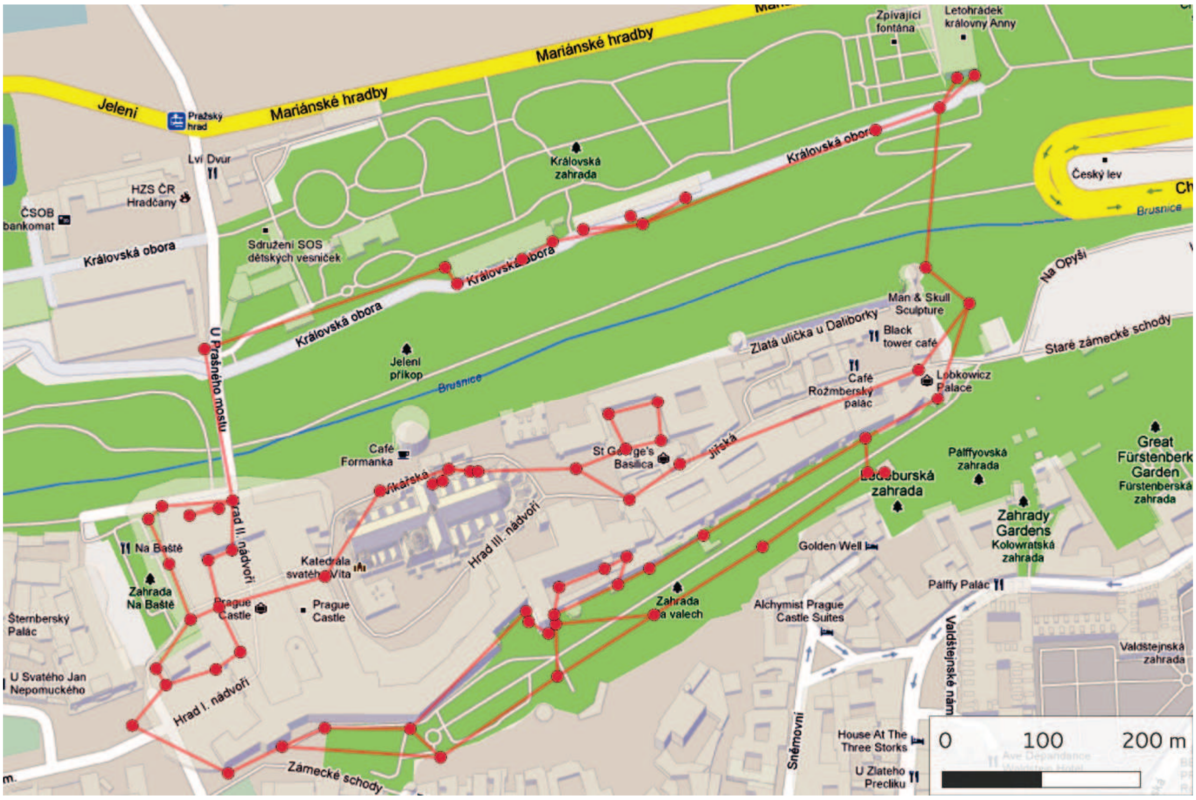

Figure 1 Scheme of the height geodetic network in the Prague Castle area

3 NEW CALCULATION OF EPOCH LEVELLING MEASUREMENTS

Overall, there were 18 epochs re-processed and re-adjusted, all of them measured between years 2004 and 2012. Original intention of the measurements was not to carry out assessments, but monitoring of individual objects. Monitored objects changed during the years and in different epochs unequal sets of points were measured. Because of the relative solution of all monitoring, the measurement onto the stable points outside the Prague Castle area was not performed, and therefore none of the points can be considered to be stable.

Processing was made with regard to these facts in epochs by the least squares adjustment, in the GNU Gama software (gama-local ver. 1.13, more in (Čepek, 2005)). As the measured values served averaged height differences measured back and forth. The a priori standard deviation was chosen to be $\sigma_0 = 0.7$ mm, measurements were conducted predominantly by the Zeiss KoNi 007 instrument.

The results of the new adjustment are relative heights of points at each epoch. For further analysis, a standard deviation of height of one point is assumed to be $\sigma_p = 0.36$ mm as the average of standard deviations of all points in all epochs. To identify the shifts between the epochs, the height difference between the epochs should exceed $\Delta H = u_p \cdot \sqrt{2} \cdot \sigma_p = 1.0$ mm for the 95% ($u_p = 2$) or 1.3 mm for the probability of 99% ($u_p = 2.5$).

4 CALCULATION OF ROBUST ANALYSIS

The results of the adjustment are relative heights of points at each epoch. It is not possible to consider any of the points to be stable, therefore the transformation with redundant measurements was chosen for the analysis and calculated with use of the robust estimation, which is highly resistant against the outlaying (here shifted) values.

4.1 THE BASIC PRINCIPLE OF A ROBUST CALCULATION

Robust adjustment methods are mostly based on the principle of maximum likelihood method and their basic property is (compared to in geodesy widely used the least squares method) high resistance against the influence (against) of outlying measurements. The principles and derivation of least squares and robust methods can be found in (Štroner, 2011). Most practically usable robust methods are based on adjusting of the weights in the calculation method of least squares (reweighting), such a calculation is then relatively easy. Methods are presented in (Štroner, 2011) too. The calculation procedure of iterative adjustment is based on the assembly of normal equations in the form:

$$\boldsymbol{A}^T \boldsymbol{P} \boldsymbol{A} \boldsymbol{dx} = \boldsymbol{A}^T \boldsymbol{P} \boldsymbol{l}' \ , \qquad (1)$$

$$\boldsymbol{dx} = \left(\boldsymbol{A}^T \boldsymbol{P} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{P} \boldsymbol{l}' , \qquad (2)$$

where $\boldsymbol{A}$ is Jacobi matrix, $\boldsymbol{P}$ diagonal matrix of weights (on the diagonal are measurements' weights $P_{ii} = K/\sigma_i^2$), $\boldsymbol{dx}$ increment vector of unknowns, $\boldsymbol{l}'$ vector of reduced measurements. Robust weight change:

$$\boldsymbol{dx} = \left(\boldsymbol{A}^T \boldsymbol{P} \boldsymbol{W} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{P} \boldsymbol{W} \boldsymbol{l}' \ , \qquad (3)$$

where robust weight change is determined by the equation:

$$\boldsymbol{W} = diag\left(w_1, w_2, ..., w_n\right); \ w_i = f\left(v_i, \sigma_i\right), \qquad (4)$$

where corrections are determined by the equation

$$\boldsymbol{v} = \boldsymbol{Adx} - \boldsymbol{l'} \ . \tag{5}$$

Robust changes are derived from the standard deviations of measurement and corrections obtained in adjustment. Various methods of calculating of the changes in weights can be used, derived on the basis of expected probability distribution of deviations from the normal distribution. Here it is worth mentioning Huber method (described in (Štroner, 2011)). When creating a robust estimator Huber came out from the normal random variable distribution. His solution is based on the replacement of the edge parts of the normal probability distribution by the Laplace distribution (a special form of the exponential distribution), which leads to greater probability of outlaying measurement on the distribution's edges. For purposes of the analysis, a L_1 norm was used, which, as a function of the probability distribution, uses directly Laplace distribution, which has in comparison to a normal distribution a greater probability of outlaying measurements occurrence. For homogeneous measurement (measurements with the same standard deviation) is a robust weight change given by the function

$$w_i = 1/|v| \tag{6}$$

and there is no need to know the standard deviation. The calculation is done iteratively, corrections used to calculate robust weights' changes are always used from a previous calculation. More to the calculation procedure is in (Štroner, 2011).

4.2 THE PROCEDURE OF CALCULATION USING THE L1 NORM

The individual epochs were adjusted and for determining the points, where between two epochs i, j were shifts, it is necessary to transform the matching points of the epoch j to epoch i. The equations of linear transformation:

$$\boldsymbol{X_i} = \boldsymbol{MRX_j} + \boldsymbol{T} , \tag{7}$$

where $\boldsymbol{X_i}$, $\boldsymbol{X_j}$ are vector of the coordinates, $\boldsymbol{M}$ matrix of the scale coefficients, $\boldsymbol{R}$ rotation matrix, $\boldsymbol{T}$ translation vector. There is only a one-dimensional transformation (only heights) needed, scale between the epochs does not change and therefore the transformation equation for heights $\boldsymbol{H}$ between epochs i and j degrades as follows:

$$\boldsymbol{H_i} = \boldsymbol{H_j} + \boldsymbol{T_{i,j}} \ . \tag{8}$$

When calculating the relationship between the two epochs, it is determined only by height shift $T_{i,j}$, and there is the average height difference between the epochs:

$$T_{i,j} = \frac{\sum_{k=1}^{n} H_{k,i} - H_{k,j}}{n} \ . \tag{9}$$

Ideally, this shift will exactly suit to all points, though practically it does not, and therefore for every point n = 1 .. k can be calculated corrections:

$$v_n = H_{n,i} - \left(H_{n,j} + T_{i,j}\right) \ . \tag{10}$$

These corrections contain a component of measurement inaccuracy, and if there was a height change, this influence too. Mean as a method corresponds with the least squares method, and in the case of outlaying measurements, here shifted points, fails to give proper results. For these reasons, it is advisable to use a robust method, which does not have such a property. The height difference between the two epochs is determined by an iterative calculation of the weighted average, where the weights are calculated on the basis of corrections from previous calculation (m-th iteration).

$$^{(m+1)}T_{i,j} = \frac{\sum_{k=1}^{n} \left(H_{k,i} - H_{k,j} \right) \, ^{(m)}w_k}{\sum_{k=1}^{n} {}^{(m)}w_k} \, . \tag{11}$$

The individual epochs were not measured at regular time intervals and also measured points changed, so the procedure has been used where at the selected epoch (namely 10) were gradually transformed all the others. As the reference epoch was chosen epoch 10, because most points were measured in this epoch, both in initial and especially in the terminal epochs. The calculated shift $T_{i,\,j}$ is not important, significant are individual corrections signalling shifts of the point between the epochs.

4.3 CALCULATION RESULTS

The calculation results are determined corrections after the transformation, which can be interpreted as deviations of individual points from the common state from a common level.

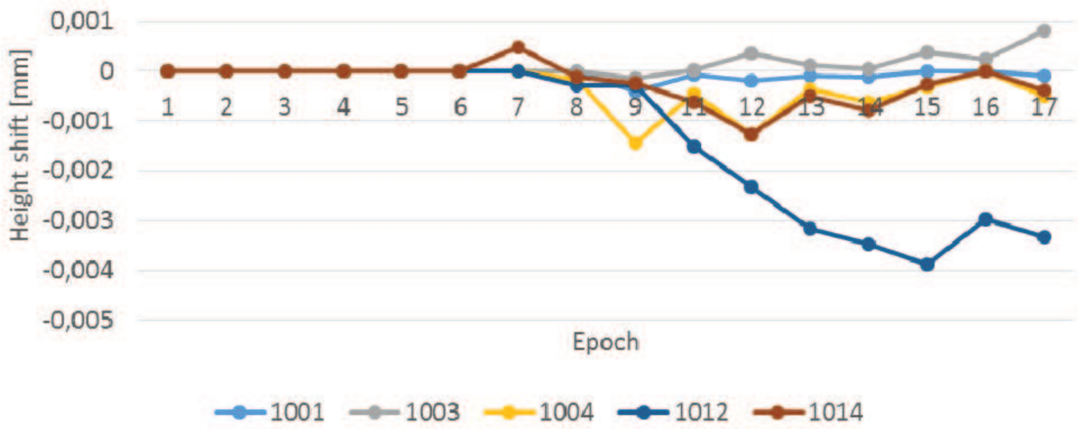

Figure 2 Example of the relative points shifts

When plotted on a graph, these corrections can give an idea of the movement of individual point between epochs. Because of the large number of points it is not possible to show all of it, an example is on the Fig. 2. Zero shift means, that point was not measured in the epoch. On Fig. 3 there are characteristic points marked by the arrow characterizing its' relative shift, grey dots marks points considered to be stable.

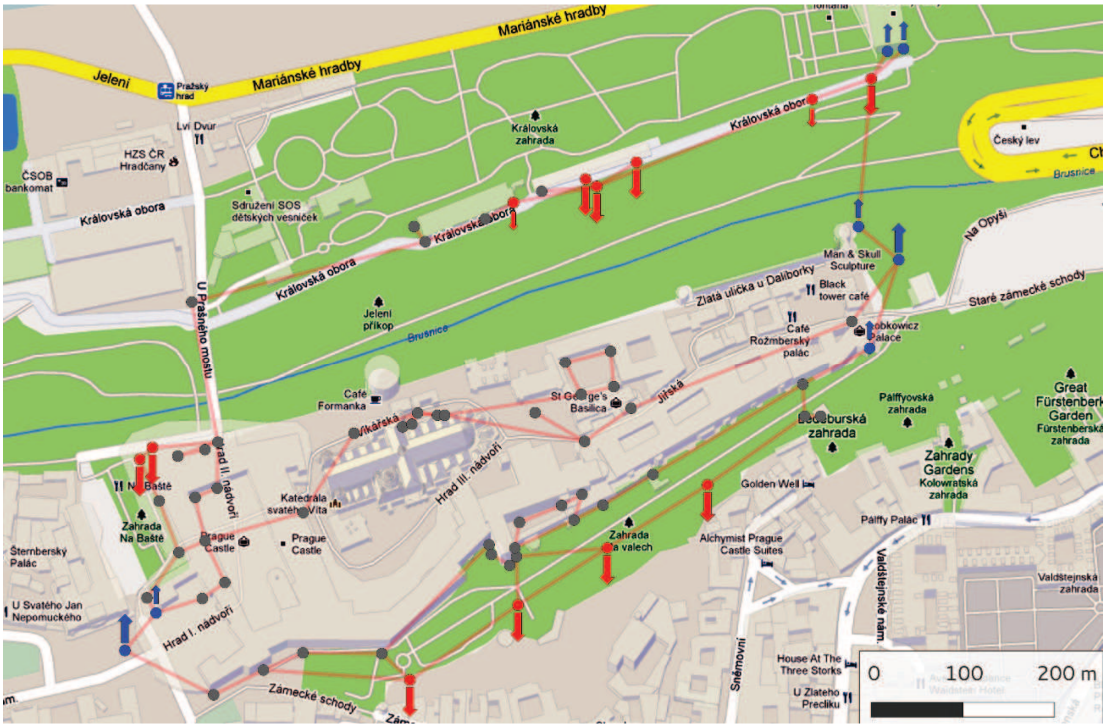

Figure 3 Relative points shifts

5 CONCLUSIONS

As a result of the new evaluation of the deformation measurements at the Prague Castle, the scheme of relative shift points was created, which is shown in Figure 2. A new methodology was used for processing the results, which involves the use of robust estimation, namely the L_1 norm. The results are consistent with observed phenomena in the field and so the presented methodology can be considered to be appropriate.

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REFERENCES

Prague Castle, cit. 3.3.2014. http://www.hrad.cz/en/prague-castle/prague-castle-tourist-information/visit-of-prague-castle.shtml

ZÁLESKÝ, J., CHAMRA, S.: Optimalizácia geotechnických konštrukcií: Projekt sledování technického stavu historických budov. In: Optimalizácia geotechnických konštrukcií, 18. – 19. září 2001, SvF STU Bratislava, Slovenská Republika, ISBN 80-227-1545-X. s. 337–341

PROCHÁZKA, J., JIŘIKOVSKÝ, T., ZÁLESKÝ, J. et al.: Stabilita historických objektů. 1. vyd. Praha: České vysoké učení technické v Praze, 2011. 229 s. ISBN 978-80-01-04776-7.

ČEPEK, A.: GNU Gama 1.9 - Adjustment in geodetic networks. Edition 0.19, 2005

ŠTRONER, M., HAMPACHER, M.: Zpracování a analýza měření v inženýrské geodézii. 1. vyd. Praha: CTU Publishing House, 2011. 313 s. ISBN 978-80-01-04900-6.